Reconstructions of science

Schmeikal, Bernd

Veröffentlichungsversion / Published Version
Arbeitspapier / working paper

Empfohlene Zitierung / Suggested Citation:
Schmeikal, B. (1998). *Reconstructions of science.* (Reihe Soziologie / Institut für Höhere Studien, Abt. Soziologie, 33). Wien: Institut für Höhere Studien (IHS), Wien. https://nbn-resolving.org/urn:nbn:de:0168-ssoar-221918

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Reihe Soziologie / Sociological Series No. 33

Reconstructions of Science

Bernd Schmeikal

Reconstructions of Science

Bernd Schmeikal

Reihe Soziologie / Sociological Series No. 33

November 1998

Institut für Höhere Studien
Stumpergasse 56, A-1060 Wien
Fax: +43/1/599 91-191

Bernd Schmeikal
Phone: +43/1/7146739

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Die **Reihe Soziologie** wird von der Abteilung Soziologie des Instituts für Höhere Studien (IHS) in Wien herausgegeben. Ziel dieser Publikationsreihe ist, abteilungsinterne Arbeitspapiere einer breiteren fachinternen Öffentlichkeit und Diskussion zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autorinnen. Gastbeiträge werden als solche gekennzeichnet.

Abstract

In four reconstructions it is attempted to lead the natural and social science debate of the basis concepts of space and time in common. For this we need a new mode of science discourse which has already been initiated in Alfred North Whitehead's philosophy of nature. In social science we reconsider the basis themes of social phenomenology, structuralism and interactionism as far as those contribute to a space-time topic. Investigations of prehistorians, egyptologists and ethno-mathematicians are of importance where we demonstrate that our concepts of space and time represent cultural institutions of meaning which on their part constitute society and require that we constantly reconstruct them. The fourth reconstruction deals with the space-time of postmodern theoretical physics and is founded on the integrative instrument of the theory of geometric Clifford algebras. We show that and how the inner symmetrics of matter are connected with the outer symmetries of space-time and that Gell-Mann's metaphor of the *"eightfold path"* that he used to denote part of the standard model of physics cannot be interpreted as quirk, in opposition to his own intention. The factor $[D_4]^m$ in the Dirac group of any geometric Clifford Algebra $Cl_{p,q}$ represents a ground template (or archetypal structure) for both orientation and logic and corresponds therefore with an interface between matter and mind.

Zusammenfassung

In vier Rekonstruktionen wird versucht, die natur- und sozialwissenschaftliche Diskussion der Basiskonzepte von Raum und Zeit zu vereinen. Dazu bedarf es einer neuen Diskursform, die bereits in Alfred North Whiteheads Naturphilosophie anklingt. Auf sozial-wissenschaftlicher Seite besinnen wir uns grundlegender Themen von Sozialphänomeno-logie, Strukturalismus und Interaktionismus. Fragestellungen von PrähistorikerInnen, ÄgyptologInnen und Ethno-mathematikerInnen werden wichtig, wo wir zeigen, daß unsere Konzepte von Raum und Zeit kulturelle Institutionen der Bedeutung sind, die ihrerseits Gesellschaft konstituieren und konstanter Rekonstruktion bedürfen. Die vierte Rekonstruktion greift die Frage der theoretischen Physik auf und stützt sich auf das integrative Instrument der Theorie der geometrischen Clifford Algebren. Wir leiten ab, daß und wie die inneren Symmetrien der Materie mit den außeren Symmetrien der Raum-Zeit verbunden sind und daß die Metapher vom *"achtfachen Pfad"*, die Gell-Mann für einen Teil des Standardmodells verwendete, entgegen seiner Auffassung nicht als Witz zu verstehen ist. Der Faktor $[D_4]^m$ in der Dirac-Gruppe jeder geometrischen Clifford Algebra $Cl_{p,q}$ bildet eine Grundstruktur von Orientierung und Logik ab und korrespondiert daher mit einem Interface zwischen Geist und Materie.

Keywords
Space, time, time-reversal, orientation, Clifford algebra, Dirac equation, standard model, phenomenology, bioenergetics, ethnomathematics, structuralism.

Schlagworte
Raum, Zeit, Zeit-Umkehr, Orientierung, Clifford Algebra, Dirac Gleichung, Standardmodell, Phänomenologie, Bionenergetik, Ethnomathematik, Strukturalismus.

Bernd Schmeikal holds a doctoral degree in philosophy from the University of Vienna, where he studied mathematics, physics, sociology and ethnology. He worked in the early High Energy Physics group (HEPHY) under Professor Walter Thirring and was Assistant Professor in sociology to Professor Robert Reichardt. He has been a postgraduate researcher at the Institute for Advanced Studies, Vienna. Currently, he is a Research and Teaching Fellow in the Sociology Department at the University of Vienna and Director of the Biofield Laboratory, a Research & Development Institute into innovative science discourse. He is author of several publications on subjects including psychology, sociology, anthropology, mathematics, physics, biology, bioenergetics and innovative medicine. He is a reviewer of the American Mathematical Society.

Contents

Foreword

A preface, a foreword, and an introduction all come in the first pages of a book before the main contents. An introduction is usually longer than a preface or foreword. A foreword is sometimes more informal than the preface or introduction, or written from a more personal point of view (Longman Dictionary of Contemporary English). It is exactly that more personal point of view which has led my considerations in that foreword.

What B. Schmeikal is developing in the following pages in terms of a reconstruction of space-time and harsh criticism on the actual and divergent interpretations of time in sociology and in physics is, so to say, a revolutionary attack against traditional thinking the roots of which are going back to the early beginnings of modern mechanics and which plays its part both in the social sciences and in physics. Since many years — presumably since the end of the Seventies — we have been talking together about the „problem" of time in sociology. What we have agreed upon very early is that time and space are fundamental categories in sociological theories but sociologists have neglected that crucial fact notoriously.

Every theory of action, be it inspired by M. Weber or T. Parsons, every theory of social structure which tries to grasp the phenomenon of social change, all have time as a latent dimension in their conceptions but never make its role explicit. Somewhat different is the situation when we look at phenomenological sociology or symbolic interactionism. The constitution of time by inner experience has a sound tradition which started from the first considerations of E. Husserl, H. Bergson and A. Schütz. These time conceptions subsumed under the term „subjective time", however, stand in opposition to what has been called „objective time" or „Weltzeit". Objective time is taken as given and selfevident in these theories and not reconstructed. Whenever A. Schütz and Th. Luckmann, for instance, speak of Weltzeit, the meaning of the term is derived from notions in the traditional understanding of physics. As a consequence, sociological theories use „second hand" conceptions of time.

Additionally, the concept of space is similarly neglected in these theories. Only the phenomenological reconstruction of the everyday world (A. Schütz) takes into account the role of space, but looks at it as a given dimension without trying to reconstruct the notion of space itself. Moving my hand is a process I can observe „outside in the space", as A. Schütz said, and simultaneously look at it in the dimension of my inner experience. Movement comprises a double quality: the one is being part of a material outside world and the second is being part of an absolutely individual cognitive experience. That difference, we both B. Schmeikal and myself were convinced already some years ago, is the expression of an epistemological and unresolved problem in social science theories embedded in the matrix of orientation and space and movement and time. Contrary to A. Giddens or P. Bourdieu, the reconstruction of space and time in social science theory can not be based exclusively on the background of everyday practice in the social world of people. The studies of P. Bourdieu show very convincingly that a time order (of the people in the Maghreb) emerges from their cultural and social practices, however, they show convincingly as well that competing time orders cannot emerge from identical practice. The reconstruction of time and space in the social siences needs also the anthropological and the historical dimension (which has been also postulated by G. Dux). It is especially that question which B. Schmeikal has put to the fore and tried to show that we are misled by using our western societies´ standards in answering problems which are of basic importance for the existence of people in general.

The notions of a non temporalized space, the concept of orientation which refers to a „joint venture" between anthropology and mathematics and, as a consequence, the necessity for refining social theory, make us aware that social theory development has come to a turning point where completely new paths have to be found which allow to leave the old ways. One of the „new" paths means putting the cognitive preoccupation and the dichotomy between individual and society (or action and structure) aside and see body, space, and movement, and experience, cognition, and time as integrating parts of an ever structured and ever structuring unique process. Only within such a conception does it make sense to say that time comes from space being aware of that space is in turn the product of a fundamental mechanism: orientation.

How could be outlined what B. Schmeikal's solution is? He has tried in his reconstructions to discuss the basic concepts of space and time in the social and natural sciences in parallelism. Such an operation needs a philosophical foundation. On the side of the natural sciences and natural philosophy the source is A. N. Whiteheads natural philosophy, conceived in 1919 in his „Tarner lectures"; on the side of the social sciences the approach of A. Giddens (The Constitution of Society) has served as a similar starting point. However, to understand his conceptions some understanding of other traditions is necessary which are linked to A. Giddens Structuration theory: Symbolic interactionism (E. Goffman), psychoanalytic theory (S. Freud, E. Erikson), social phenomenology and existential psychology (R. D. Laing), structuralism (C. Lèvi-Strauss, J. Lacan) as well as the philosophical and social-scientific discussion about the relative importance of some basic questions of structuralism (R. Boudon, G. Deleuze). The sometimes heard judgment that these approaches and questions were outdated is completely wrong. Even contributions from the works of prehistorians (M. E. P. König, A. Leroi-Gourhan), developmental psychologists (J. Piaget), mathematicians (D. Hestenes, P. Lounesto), and especially ethno-mathematicians (P. Gerdes, M. Ascher) have proved to be useful. Thus, one of the insights of B. Schmeikals effort is that constructing social science concepts of space and time is an interdisciplinary work.

Some people are still convinced that in the natural sciences there had been reached a level of exactness which would never be approached by the social sciences because of their phenomenological, interactionistic, and structural character. However, it is a misconception because of one fundamental reason. Physics and the natural sciences in general have not reached that exactness, either. What is often misunderstood as exactness is — by reasons which are to be found in the history of those sciences — a totalitarian mediaeval style of knowledge, which has been used to make something plausible which does not exist. The natural scientists have constructed an exact space-time which is by no means able to coordinate what happens factually in nature. The social sciences have not been able to keep such a conception upright in a similar totalitarian and comprehensive manner because of the evidently phenomenological character of social relations and because of the fact that they have not been preshaped in their epistemology by scholastic doctrines. It is very interesting to see that the history of the social sciences is reporting nothing similar to the clerical arithmetics — part of the history of the natural sciences — by which monasteric and social life have been organized for centuries.

A. N. Whitehead has made it very clear that there is no correspondence between the absolute space-time conceptions of the mathematical physics stemming from the extensive abstraction of a natural phenomenology and reality. Geometrical space and serial time are 'nonentities', which appear as limited measures of quantified serial events only within a certain intellectual construction. They belong — to speak from a post-modern point of view — entirely to the symbolic discourse. The movement of natural events does not stick to them. A. N. Whitehead has put it in the following words: Nature gives something to thought which is for thought only. The presence can not be serially separated from past and future as it is the case in physics. The presence is vivid presence which is permanently integrating the past and the future.

Natural change is becoming evident in the vivid presence in the quality of sensual awareness. Events are not combined in terms of space-time but by relations of extension. It is these relations from which the intellectual discourse is deriving the concepts of absolute space and time. The extension of events appears in the sensual awareness and in co-presence. As long as we think about natural events the term sensual awareness suffices; in social phenomenology we use the term co-presence additionally (A. Giddens). Co-presence signifies the mutual awareness of actors in the qualities of sense awareness, cognitive and practical consciousness, and body awareness. Representation in nature as well as in society rests on the integrative intelligence of awareness. The material basis of that kind of intelligence is within people the body awareness. The institutions of significance depend on that form of awareness. There is no reason why the extension of the presence should be separated from the extension of awareness and that, in turn, from nature. Nature can be seen in the same phenomenological way as society.

Space and time undergo permanent reconstruction. The space of physics needs reconstruction as well as social space; the same holds true for their concepts in terms of symbolic and discoursive forms. The concepts are to be seen as extensions of structures of meanings from the social into the natural sphere. Obviously, the meaning/significance of space does not end where our cognitive constructs end. There is no need to stop our efforts as social scientists at the doors of physics. The significance of the space of nature is not outside our life. It is part of nature, and exactly therefore, it is accessible for human consciousness.

If the reconstruction of the space-time concept is discussed, it is obvious that we do not speak primarily about Euclidean space or even space of Minkowski. Most of us would not be able to understand the concept as a whole. In its comprehensiveness it could be reconstructed and understood probably only by a small elite. Were there not a group of mathematicians working with Clifford algebra in permanently reproducing most recent knowledge on space concepts and number-fields, a lot of relevant knowledge would be lost soon. Besides the problem of measurability, we can state that the space concept has other parts which form a unique system of meaning and significance.

Orientation is such a part of the space concept and, therefore, one can speak of the orientation concept and of the mathematics of orientation symmetries. The orientation concept is part of the consciousness of every human being. It is interesting to keep in mind that in segmented societies there are a lot of different space concepts which differ quite considerably from the concepts in mathematical physics. In the sand-drawings of the Tshokwe it is, for instance, usual to represent 'co-presence' of human beings, animals, and cultural artefacts in — so to speak — a bioenergetic whole in one uninterrupted drawing ('monolinear Sona'). In his second reconstruction B. Schmeikal shows that in all these narrativ-ideographic representations the orientation concept is working as a principle of structuration. Such a concept may also be the basis of what in developmental psychology has been called the operational structures of thinking. Thinking is connected with the societal institutions of space concepts; therefore, A. N. Whiteheads question for the connection between thinking and space can be answered. He himself expressed his point in the following words:

"The connexion of thought with space seems to have a certain character of indirectness which appears to be lacking in the connexion of thought with time". (A. N. Whitehead 1964, p. 37) But, what about the proposition that nature would be in space or space in nature? A. N. Whitehead says: "Matter, in its modern scientific sense, is a return to the Ionian effort to find in space and time some stuff which composes nature. The whole being of substance is as a substratum for attributes. Thus time and space should be attributes of the substance. This they palpably are not, if the matter be the substance of nature, since it is impossible to express spatio-temporal truths without having recourse to relations involving relata other than bits of matter. I wave this point however, and come to another. It is not the substance which is in space, but the attributes. What we find in space are the red of the rose and the smell of the jasmine and the noise of cannon. We have all told our dentists where our toothache is. Thus space is not a relation between substances, but between attributes". (p. 21)

The concepts of orientation and space which are discussed by B. Schmeikal are means to reconstruct social space. In a certain sense, they are bioenergetic realities. They are real in the way that they can be represented geometrically and that they offer some mathematical lawlike rules; however, in fact they are symbols, signs and sentences within a language spoken and drawn which needs permanent reconstruction (gesprochene und gezeichnete Erzählsprache). It can be seen easily that the life styles belonging to those culturally specific concepts of space give stability in the conduct of behaviour. In all cases natural and social space build a whole.

Anton Amann

To

Ronald David Laing,
who,
when I was young,
saved my life by a theory.

Refining Social Theory

- First Reconstruction -

1 Prologue

In a recent train of thought we have reconstructed the concepts of space and time and tried to go beyond the traditional difference between natural and social science.[1] In order to better understand those concepts we have to refine social theory. By a refinement I mean to make it finer by a deconstruction and a reconstruction. By *social theory* I refer to the notion used by Anthony Giddens in his *theory of structuring*. In this way we locate the European social theory in a domain of intersecting theoretical approaches such as symbolic interactionism, structuralism and poststructuralism, phenomenology and so on, and also mark the limits of methodological individualism.

Social theory must first be deconstructed because it contains an insufficiency resulting from a fragmentation of the institutional order of signification. It is often believed that only symbolic and discursive forms in thought or spoken language carry meaning, a reduction resulting from the old theory of coding. So the body is excluded from the institution of signification, and it is not elucidated clearly that awareness goes beyond consciousness. The awareness of actors can activate cognitive consciousness, practical consciousness and the subconscious, that is, bioenergetic or body awareness. The old Ansatz is based on a fragmentation of meaning which is neither productive nor necessary, but it reckons up a subconscious state of violence by taking its effect unnoticed. After eliminating this effect we reconstruct the social theory by essentially following the old stream of insights which constitute its significance.

2 Institutions of Meaning

Trying to overcome false theories which were organized alongside the subject/object-duality, social theory has given up the belief that memorization was a mere regaining of information about past events. It introduced perception as action integrated with body motion in space and time The basic mechanisms of memory were thus identified with patterns of anticipation by which the past is transformed into the future (Giddens 1984, ch. 2): *"Perception is organized via anticipatory schemata whereby the individual anticipates new incoming information while simultaneously mentally digesting out. Perception normally involves the continued active movement of the eyes, and usually of the head, even when the body is at*

[1] This was begun in a seminar on "Time in sociology and in physics" held by Anton Amann and me at the Institute of Sociology, University of Vienna.

rest. Because schemata are anticipations, they are, as one author puts it, 'the medium whereby the past affects the future', which is 'identical with the underlying mechanisms of memory'" (p. 46 mentioning Cohen 1979*)* In active presence the actor is moving his body and thereby coordinating physical and social space while structuring time. Moving the body and/or parts of it, his/her awareness is shifted, interupted, concentrated or extended so that there is a serial order of absence and presence, concentration and deconcentration. As Schütz has pointed out, while the attention is diverted from the object of presence, we experience time. Putting it radical: time is a loss of presence. Also, we cannot speak or write about the presence without sinking back into the past, it was said.

As soon as we understand this whole approach we must also accept that the body is part of what was called the *institutional order of signification.* Symbolic order, discursive forms and body signification are forms of institutional order. To relate a *discursive consciousness* to some other says nothing other than that we articulate something by words. *Body signification* denotes the fact that we transform and express meaning by body language. What we mean by signifying something in body language might be known to us or not. It may be accessible to consciousness or not.

In "The Constitution of Society" Giddens has shown very clearly that the *knowledgeable* appears as a partition of three territories: *discursive consciousness, practical consciousness* and the *subconscious* Social life means that those three are in a vigilant interaction. Now we observe that the domain of that which seems meaningful to the actors — the extension of signification —' is not invariant, but variable. It depends on each actor's active presence in those three parts What is meant by an *active presence?* It is meant the bioenergetic reality of both cognitive and body awareness. Those actors who feel at home in all three territories, who are actively aware of the reactions of their discursive consciousness, practical consciousness and the unconscious, experience a maximum extension of presence, a meaning which seems to cover the whole of their existence, but they experience no separation of meaning Such a conception of presence is necessary in order to be able to obtain a precise notion of power, especially religious power. All religious power is rooted in belief systems on time and total presence. Total presence signifies the mystic divisions of world religions Beliefs on time constitute the normative core of the order of worldly life.[2]

Thus it is evident that what is theory and what is not theory depends upon the underlying partition of signification In the witness of complete integration of meaning a theory is no longer theory, because the perception of *symbolic* meaning is no longer separate from the meaning of body experience. Then the signification of signs of language has no other meaning than the signs of body language, but they are of the same quality. This quality is disclosed by inner experience. For actors who have a social theory this signifies states of extended awareness where actors and theory are one. Then the sociologist is his theory. He

---

[2] Be aware of the ecclesiastic arithmetics "computus" used to organize medieval monestary life. (Borst, 1990)

is an embodyment of the theory, and therefore there is no theory at all. In addition, such sociologist cannot be conceived as different from other actors, but, in a concrete way, represents them all. So he is no sociologist either. This extreme state, which turns into a paradox when conceived in cognition alone, is only one side of a rope the other of which represents a bioenergetic fragmentation of meaning which can best be explained by saying that bioenergetic spatial separations in the muscular system tend to be projected onto cognition so that separations are transformed into distinctions. Clearly, in such a case where body language is fragmented most, the actor is yet convinced that all the distinctions he makes are valid. His world, however, decays into bits of sense and nonsense.

Having thus deconstructed the current Ansatz, social theory can be reconstructed by essentially the same stream of insights — phenomenology, interactionism, linguistics and so forth — by which it was originally formed.

3 Social Phenomenology and Bioenergetics

On two occasions Giddens quoted R. D. Laing (1971) in connection with experiences of hysteria and psychosis. In some other monography Laing has also given a brief and useful definition of *social phenomenology*. He wrote:

"Social phenomenology is the science of my own and other's experience. It is concerned with the relation between my experience of you and your experience of me. That is, with inter-experience. It is concerned with your behaviour and my behaviour as I experience it, and your and my behaviour as you experience it. This is the crux of social phenomenology. Natural science is concerned only with the observer's experience of things. Never with the way things experience us. That is not to say that things do not react to us, and to each other. Natural science knows nothing of the relation between behaviour and experience." (Laing 1975, p. 16f)

By the time R. D. Laing was forced to go deeper into *"The Facts of Life"* (title of his book first published in 1976) where he figured out intrauterinal life as an original locus of emotional coordination where patterns of emotional experience are formed which are fundamental for the structuring of our postnatal social life. A profound sense of touch coordinates our first universe in accordance with the psychogenetic patterns of orientation (Jungs Mandala, see also Laing and Caretti 1981, forth chapter: *About Jung*). Our first beloved is our placenta. In his great theoretical work *"The Voice of Experience"* (first published 1982) Laing demystifies the frontiers between phenomenology and objective science in his very first sentence: "*Experience is no objective fact. A scientific fact need not be experienced.*"

In his chapter on the unconscious, time and memory Giddens (1984, p. 46) presumes that "it may very well be that touch, ordinarily regarded as the most humble of the senses and certainly the least studied, provides most clues for understanding perception in general". From those of Laing's writings unquoted by Giddens it is entirely clear that this presumption is to be taken seriously. It is directly connected with the whole field of medical bioenergetics as has been developed by Reich, Boyesen, Lowen, Boadella, Pierrakos and others. When bioenergy interpenetrates the body and thereby expends from the inner (core energy) to the outer, it activates our body awareness which involves both an inner sense of emotional flux and a peripheral sense of touch which, however, extends beyond the skin and is coupled to the outer. Thus, by moving our bodies through physical space — and this space is no less social than it is physical and it is no less physical than it is social — we experience social space as a bioenergetic emotional reality, be it subconscious or not, which allows us to coordinate and to transform our inner and outer reality thereby giving spatial orientation and serial order to our actions.

From this Ansatz too there follows that presence is not a temporal modality, but that it is an extension in space. Further it involves a permanent calibration of orientation. While moving our bodies through space we give it an orientation. By moving the body foreward or backward, looking to the left, to the right, up or down, we perform certain spatial operations in a serial way A•B•C• . . . and their *product* may or may not commute, that is, generally $A{\bullet}B \neq B{\bullet}A$ which is fundamental for the experience of irreversibility. Thus our movements in social space give rise to temporalization. As presence is a bioenergetic reality, it promotes interaction. Social interaction, in fact, stimulates, triggers, concentrates, shifts, weakens or amplifies, rotates or dilatates our bioenergetic sensual field. This field has an *"intension"* (in contrast to extension) in the energetic system of the body (the energy channels of acupuncture, nerves, mitochondria and so on). It can be in balance or out of balance, may have a high or a low degree of symmetry, may be stable or fluctuate. It shows both physical and social behaviour. That is, it can be measured by certain measurement devices of electromedicine and is at the same time subject to social interaction and therefore reacts to our experience of the behaviour of ourselves and others. Bioenergetic phenomena are accessible to both natural and social science. As is well known from bioenergetic research work, traumatic experience and psychological qualities of interaction in general are stored up in the deeper regions of muscles. In this way social structuring is transposed onto bioenergetic structuring in the body. The body appears as a map of social interactions. Therapy, in the context of bioenergetics (Lowen) or biodynamic massage (Boyesen) aims at a removal of muscular blockades and the accompanying free flow of emotions. Such removal of blockades is paralleled by a delivery of subconscious contents stemming from childhood. There is no transformation of the bioenergetic system without changing cognition and psychological dispositions as a whole. Delivery from all blockades — if only for some period of time —brings forth a radical change of inner experience as well as our inner sense of time. Presence is usually deepened and serial time looses its significance. This is surely a reason why people who experience such bioenergetic changes — which involve a change of character, indeed — develop a sense for mythology. Large parts of Laing's analysis of prenatal experience are organized along certain myths (e. g. *"The Facts of Life"*, ch. 5: Life before Birth). Laing points to mythology in order to show that our emotional life has a continuation and structure in physical space (ch. 4: *Feelings and Physics*) which is extending in social space. It is surprising that Giddens discussing the active organization of space (*"spacing"*) following Goffman (and even Erikson) explains the meaning of front- and backside in communication, but does not realize the importance of orientation of space as a whole: to the left, to the right, infront and behind, below and above, and last but not least, their transposition onto inner and outer. Neither in social theory, nor in modern mathematical physics have we become aware of the significance of orientation. But we shall see that the institutional order of signification, which is known to be given by symbolic and discursive forms, is indeed resulting from an incorporation and socialization of orientation. Orientation in social space helps us to transform the past into the future.

Where social theory speaks of the most general mechanisms of memory in terms of patterns of anticipation coordinating our movements through space, we can be sure that the most basic of all those templates of experience is orientation. This template serves as an interface between physics and social reality. Orientation in both physical and social space is one thing. Mathematically, it can best be represented at the level of algebra which is the reason why it has very fine methodological consequences which affect both physics and social science.

4 Structures and Structuring

Wherever there is constitution of meaning there is *"structuring"*. It may happen anywhere — in the womb, in the muscles, in the central nerveous system, in a firm, in a school, in religion, in language, in a theory and so forth — and the word is entirely unproblematic. Yet, the idea of structure, as Raymond Boudon opened his discussion already in 1968, undoubtly ranges among the most obscure key concepts of social science. "It raises the question whether the outstanding contributions of certain *'structuralists'* are not *'the product of their genius rather than the outcome of their method'*, as Leibnitz said of Descartes' analytical geometry. These difficulties are acknowledged by the majority of authors who have attempted, individually or collectively, to analyse the concept of structure" (Boudon 1971, p. 1). Giddens is aware of this when he points out that 'structuration' is not a beautiful notion (p. 30). Yet, he uses it in the subtitle "*Outline of the theory of Structuration*" and says that he had found no appropriate word.

Gilles Deleuze has listed a number of criteria to realize structuralism. As the sixth he denotes the void locus: "Ein Schüler Lacans, André Green, verweist auf das Vorhandensein des Taschentuchs, das im Othello zirkuliert, indem es alle Serien des Stückes durchläuft. "Die Art dieses Objekts wird von Lacan präzisiert: Es ist immer im Verhältnis zu sich selbst verschoben. Es hat die Eigenschaft, nicht dort zu sein, wo man es sucht, aber dafür auch gefunden zu werden, wo es nicht ist. Man kann sagen, daß es 'an seinem Platz fehlt' (und von daher nichts Reales ist). Ebensogut, daß es sich seiner eigenen Ähnlichkeit entzieht (und von daher kein Bild ist) — daß es sich seiner eigenen Identität entzieht (und von daher kein Begriff ist). "[...] was versteckt ist, [ist] immer nur das [...], was an seinem Platz fehlt, wie sich der Auftragszettel ausdrückt, wenn ein Band in der Bibliothek verlorengegangen ist. Und stünde dieser Band auch auf dem Regal oder im Fach nebenan, er wäre verborgen, wie sichtbar er auch scheinen mag. Das kommt daher, daß man nur von dem, was seinen Ort wechseln kann, das heißt vom Symbolischen buchstäblich [à la lettre] sagen kann, daß es an seinem Platz fehle. Denn für das Reale, in welche Unordnung man es auch immer bringt, befindet es sich immer und in jedem Fall an seinem Platz, es trägt ihn an seiner Sohle mit sich fort, ohne daß es etwas gibt, das es aus ihm verbannen könnte.' (Lacan: Schriften I, S. 24) Wenn die Serien, die das Objekt = x durchläuft, notwendig Verschiebungen darstellen, die im Verhältnis zu einander relativ sind, so weil die relativen Orte ihrer Glieder in der Struktur zunächst von dem absoluten Ort eines jeden, in jedem Moment, im Verhälnis zum Objekt = x abhängen, das beständig zirkuliert, beständig im Verhältnis zu sich selbst verschoben ist. Eben in diesem Sinne bilden die Verschiebungen und allgemeiner alle Austauschformen kein von außen hinzugefügtes Merkmal, sondern die grundlegende Eigenschaft, die es ermöglicht, die Struktur als Ordnung der Orte unter wechselnden Verhältnissen zu definieren. Die ganze Struktur wird von diesem ursprünglichen Dritten bewegt — das sich jedoch auch seinem eigenen Ursprung entzieht. Indem das Objekt = x die Differenzen in der ganzen Struktur verteilt, die differentiellen Verhältnisse mit seinen Verschiebungen wechseln läßt, konstituiert es das Differenzierende der Differenz selbst. "Wenn es stimmt, daß die strukturale Kritik die Bestimmung der 'Virtualitäten' in der Sprache zum Gegenstand hat, die vor dem Werk existieren, so ist das Werk selbst struktural, wenn es sich zum Ziel setzt, seine eigenen Virtualitäten zum Ausdruck zu bringen. Lewis Carroll, Joyce erfanden 'Kofferwörter', oder allgemeiner, esoterische Wörter, um die Koinzidenz klanglicher verbaler Serien und die Gleichzeitigkeit von Serien assoziierter Geschichten sicherzustellen. In Finnegan's Wake ist es überdies ein Buchstabe, der Kosmos ist und der alle Serien der Welt vereinigt". (Deleuze 1992, 42-47)

Also in my work, there appears a symbol in which all the series of the world meet. This is the mandala ⊕ which signifies the sociogenetic structure of orientation. It is clear that this void can be given an invariable meaning in social space which coordinates physical space with circumstances of interpersonal perception. This means the following: Suppose we are standing face to face. While my body — the body of ego — moves towards you, the body of alter, I experience that I move foreward and approach you and you experience that I move foreward and approach you. Both experience that we approach each other. Also you and I know or/and experience that my right hand is on the side of your left hand and my left hand is on the side of your right hand. Both of us share the experience — if only unaware of it — that what is above is above both our heads and what is below is below both of our feet. Most of the physical space infront of you — is behind my back and most of what is infront of me is behind your back. Yet, standing face to face, while I am aware of you — gestures, speech, eye movements and body language —most of what is in front of me is you and is therefore not behind your back. It is only when I loose attention that what is infront of my face falls into the unmeasured space behind your back. And only when our attention is reduced, in the instance, we may be unaware of what is above, yet we say, it is clear there is an above. Also we may not know what is below our toes and ankles. Yet, we say we are sure there must be something below our toes and ankles. All this is influenced by practical consciousness and by the subconscious, and that all this is as it is wherever we go and in whichever Kulturkreis, culture or society we are moving. The most basic operations of orientation form an invariant core of experience. In physical space those appear in the disguise of algebraic invariants. In mathematics they form a total structure (of algebraic group theory). In social space the experience of those invariants is temporalized through the structuration of the copresence of actors. In our presence not all of physical space is present at any time. Social space is structurated while in the actors' copresence social meaning is constituted by a synergetic arranging of absences and presences. That is, by moving eyes, face, arms, legs and the whole body, the attention af actors can be manipulated such that whole actors may for example vanish in void space. Temporalization of social structure implies decomposition of social space into bundles of absences and presences. Consider the following situations:

[1a] Consider a group of people sitting in a room and having a conversation. One actor may remain silent all the time, yet, the others feel his presence.

[1b] Consider a group of people sitting in a room and having a conversation. One actor may remain silent all the time, yet, the others feel his presence.

[2] An actor may remain silent all the time, and the others do not realize he is here.

[3] An actor may speak for a while, but the others refuse eye contacts with him until it seems to them he is not here, though they know he is here.

The first case [1a] may signify a state of bioenergetic or charismatic presence. The second case [1b] may signify the presence of a powerful actor who is bioenergetically absent. Yet, institutional history assigns to him symbolic power so that his symbolic presence as an image the others have of him can activate a bioenergetic reality in the others, which brings forth a presence in social space while he himself feels absent. Note, both cases have been articulated by the same sentence! The third case [2] may indeed signify a state of bioenergetic and interpersonal absence (nobody knows him). But in what way is the actor in the forth case absent or present? He may be bioenergetically present — he is attentive to the body language of the others — but the others don't like what he said and have established a bioenergetic blockade between them and him. There may be no flow of feelings between them and him. However, he is present as an image the others have of him. Therefore he is symbolically present. He may even be present as an effect on the discursive form — but bioenergetically he is in an unsymmetric relation: though he is bioenergetically present, the others have blocked him out and are themselves bioenergetically fragmented. This signifies a state of reduced institution of social signification. In such a situation an extreme difference between physical and social space is generated. Some actors appear to be presently absent in social space while they are physically present and absently present in the symbolic order of discourse. It seems that all combinations are possible

the actor is present

physically	symbolically	discursive	bioenergetically
1	1	1	1
1	1	1	0
1	1	0	1
.	.	.	.
.	.	.	.
.	.	.	.
0	0	0	0

The structuration of social space is based on orientation. The serial order of orientations (re)constructs a global orientation of space and thereby brings on the temporal order of action. The (re)creation of social space involves a temporalization of orientation. The most invariable mechanism of memory or ground template of anticipation is the orientation of physical space which is given by the (anti)automorphisms of its coordinates. In social phenomenology the pattern of orientation of physical space serves as a potential only. It is not actively and completely reconstructed by all the actors all the time. But it is the breaking of its symmetries and the fractionalization of the topology of space which gives rise to social space.

The concept of orientation is not restricted to microsociological institution of meaning. To arrange the copresence of actors within the intersections of spatiotemporal bundles of corporate actors, the same pattern of orientation is necessary. To travel from Europe to Japan it is needed, and even to surf in the internet you have to accept that the screen of the PC is organized along the dihedral symmetry of a void plane. The geometric features of empty space cannot be neglected. But in social space they have constituted a bifurcation point from which the institution of symbolic meaning could take off.

5 Ethnology and Mathematics

To develop exact concepts of space and time valid in different non-historic and ethnic societies such as stone age communities and peoples without linear writing, we have to understand the relation between ethnology and mathematics. This relation is best re-presented in a monography by Paulus Gerdes (1997), which I have compiled on the following pages (Schmeikal 1997). There are two views of mathematics. The first regards mathematical knowledge as an infinite record of virtual truth which can, in principle, be ascertained by any society independent of their location in social space and time. This universal truth can be understood for example in terms of Platons paradigmata or as some latent stock of statements deducible from some axioms or elementary forms at any possible time by the basic rules of mathematical reasoning and deduction (deductive theory). Thus mathematics appears as a king or queen of science which has to be thoroughly separated from any empirical science. It is, as it were, beyond culture and experience.

The other view respects the social conditions of human experience. It is aware that even mathematical ideas differ from culture to culture and therefore emphasize the concept of *"cultural anthropology of mathematics"*. Paulus Gerdes, today rector of the Pedagogic University in Maputo, Mozambique, regards cultural anthropology of mathematics as an important concept of Ethnomathematics. He mentions Wilder, White, Fettweis, Luquet and Raum, who among mathematicians, ethnographers, psychologists and pedagogicians were isolated forerunners in ethnomathematics. Wilder for example had pointed out that each human culture determines upon its constitutive elements and particularly its mathematics. So it would be productive to study the relation between culture and mathematics from this viewpoint of cultural elements. Wilder had quoted White's essay *"The locus of mathematical reality: an anthropological footnote"* from 1947, who again refers to Keyser (1930). Keyser had defended the thesis that the type of mathematics that can be found in any larger culture is a key to the specific character of this culture as a whole. In his essay White asks about the locus of mathematical reality. Does it belong to the phenomenal world or is it a mere invention of thought? He comes to the conclusion that the mathematical truths and realities are part of the human culture and that mathematics is a form of behaviour brought upon by the reactions of a specific primate organism to a group of stimuli. Thus mathematical truths appear to be both discovered and produced by the mind. Mathematical thought does not originate in Euclid or Pythagoras, nor in ancient egypt, nor mesopotamia, but it rose from a prehistoric mental condition beginning with human culture.

Wilder and White did not know the studies of the ethnologist and pedagog Fettweis (1881-1967) on early mathematical thought and culture. They also did not know Luquet's remarks on the cultural origin of mathematical ideas. Luquet was a french psychologist and it is in the paleolithic Ilé de France where the oldest ideograms of mathematical conceptions were fo-und. It also seems that "Arithmetics in Africa", a monography by Raum (1938), was not well known among contemporary mathematicians and anthropologists. This book contains the core of a lecture course held at the Colonial Department of the Pedagogic Institute of the University of London. In his preface T. Nunn says that education cannot really be effective if it is not intelligently connected with native culture and lively interests. Raum suggested that advanced arithmetic procedures had to be developed out of their own cultural origin in order that pupils may take possession of generalizations and abstractions as enduring instruments of thought.

An educational program following such an attitude has gradually been put into realization from 1976. According to Shan and Bailey (1991), ethnomathematics is a relatively new concept which has essentially been explored and publicated by two mathematicians — Ubiratan D'Ambrosio of Brazil and Paulus Gerdes from Mozambique. The unsuccessful transfer of instruction courses from the North to the South, the entanglement of mathematicians with the Vietnam war and the endeavour of the young politically independent states to get an independent education system stimulated reflections on the role and implications of mathematical research and teaching. In the ending 1970s and beginning 1980s, mathematicians became increasingly aware of the cultural and social implications of mathematical training. There were workshops, conferences and international congresses where development and socio-cultural foundations of mathematical education in the third world were discussed. Motivation to teach mathematics in a real social world was a major issue at those gatherings. D'Ambrosio, played a stimulating role in all those initiatives. According to D'Ambrosio ethnomathematics has to offer a methodology that allows to explore the origination, transaction, circulation and institutionalization of (mathematical) knowledge. In a dialogue with Marcia Ascher he classified ethnomathematics as a *"program in the history and philosophy of mathematics"*.

Various notions have been proposed to represent a contrast to the universal academic mathematics, such as native mathematics, sociomathematics of Africa, nonformal mathematics, mathematics in the African socio-cultural environment, spontaneous mathematics, oral mathematics, suppressed mathematics, non-standardized mathematics, people's mathematics, mathematics codified in know-how, implicit and non-professional mathematics. One of the most appropriate and well-aimed notions is that of a hidden or frozen mathematics coined by Paulus Gerdes in (1982, 1985). Although it is likely that most of the mathematical attainments of former peoples without writing are lost, it is possible to thaw them out by pondering over very old techniques such as wickerwork, texture and decorated carved work. All the above proposals were only preliminary. But little by little their various social pieces of mathematics were integrated under the common denominator of ethnomathematics. This process of integration was accelerated by the formation of the International Study Group for Ethnomathematics (ISGEm). Thus, ethnomathematics turns out as a joint venture of mathematics and cultural anthropology.

In 1989 Bishop asked himself: if all ethnomathematics is mathematics, anyway, why should we call it ethnomathematics and not simply mathematics of this or that (sub)culture? In agreement with Bishops annihilation, Gerdes regards ethnomathematics as a research field reflecting the awareness of the existence of many cultural forms of mathematics, and he points out that as such it has areas of contact in common with Struik's *Sociology of mathematics.* According to Crump the denotation of *"ethnoscience"* can be used to refer to a system of perception and understanding which is typical for a given culture. One should add to that that Lévi-Strauss had made it clear in his famous monography *"The savage mind"* that systems of cognition and knowledge in peoples without writing do not differ essentially from scientific knowledge in our civilized world, that is, ethnoscience is science.

Explaining the interaction of all those important contributions to the research field of ethnomathematics, in the section *ethnomathematics and didactics* Gerdes even comes into contact with Ferreira's concept who speaks of an embodyment of mathematics within the culture of a people. This idea is very close to the notation of "habitus" as is used by the french anthropologist Bourdieu and also to the theory of structuration developed by Giddens. Mathematics in this context turns out as an internalized form of cultural behaviour. For Ferreira, ethnoscience is a method to approach the notations of institutionalized science. Looking at ethnomathematics from that point of view the works of the Aschers form such important cultural links in which both ethnic and first world mathematics come together.

Taking a choice for either Gerdes' or Marcia Ascher's book is not possible in the reconstruction under our eyes. They carry the same title: *Ethnomathematics,* but the first does not use western mathematics, the other does: algebra, group theory and so forth. The second is useful, because we wish to investigate if and how ethnomathematics influences and modifies modern mathematics. This will also result in a transformation of our so called advanced concepts of space and time.

Gerdes' contribution is radical. It does not rely on any pregiven mathematical concept such as group theory, number theory or geometric algebra as are trained on any American university college But it is entirely based on collections of the empirical informations, notions, observations and attainments of knowledge in the population of Mozambique and elsewhere. Yet, it brings forth an entirely exact mathematical science.

Ethnomathematical research began in Mozambique briefly after independence in 1975. Then there were only five qualified mathematics teachers, and the elaboration of a training program for teachers was a great challenge for Gerdes and his collegues. At the same time the euphoric political climate after the liberation stimulated the search after the national cultural roots and identity. The 1978 research project on *"Empirical knowledge of the population and mathematics education"* led to a first reflection on the *"historical development of the number concept"*. To intensify the motivation of future teachers by exploration of the mathematical elements, a methodology of thawing out and waking up knowledge hidden or frozen in those cultural elements was developed. The researcher first has to become acquainted with the customary process of manufacture (e. g. plaiting) of mats, baskets, frails, eel-pots, wicker-traps and so on, and on each stage of this process he has to ask himself what kind of geometric considerations can be helpful to reach the next level. Such thawing out of very old techniques promotes reflections on the early history of geometry. The foundations of mathematics are thus recognized to originate in social action. Planning teaching on such a basis has to be seen as an act of emanzipation and liberation from outlandisch forms of culture. On a pan-African level there have been carried out several studies concerning the Pythagorean theorem and connected with *"a widespread decorative motive"*. A whole series of studies is connected with the single cultural element of a woven funnel the analysis of which led to the discovery of a new construction principle for regular polyhedra. After the peace treaty of 1992 the new field research related to a large collection of so-called Sipatsi hand bags resulted in the presentation of a catalog of decorative ribbon-designs and the mathematical analysis and didactic exploration of those patterns and their symmetries. There followed many didactic experiments on counting methods, symmetries of ornaments on baskets and window-grates and the like. The coronation of this ethnomathematical research and development work is perhaps represented by the analysis and reconstruction of mathematical elements in the Tshokwe 'Sona' tradition. Gerdes was first confronted with the Sona sand drawing tradition through Fontinha's classic work *"Desenhos na areia dos Quiocos do Nordeste de Angola"*. Skimming the book, Gerdes had the impression that it represented some unknown geometry, and this impression can be deepened and explored in precise terms by going into Gerdes' book.

It has three parts (1) a historic reconstruction, (2) a didactic and mathematical exploration and (3) comparative studies Sand drawing is part of a story telling tradition. The sona (sing.: lusona) are drawn by men women and children. With each person dying some important part of this tradition is lost Sona are line patterns which follow some geometric algorithm. To draw them, first a rectangular grid of dots is imprinted into the sand. Then a continuous figure is drawn surrounding the dots without touching them. Often there is made the specification that each line be traced only once and without lifting the finger. According to their construction principles, the drawings can be partitioned into equivalence classes. The drawing experts — the akwa kuta sona — know various rules to combine monolinear drawings into larger monolinear patterns In the first part of the book those rules are reconstructed.

In the second chapter arithmetic relations are studied that can be connected with the drawings. For instance to each rectangular array there exist related arithmetic series and symmetry elements Even the calculation of the greatest common divisor of two natural numbers can be carried out by the aid of sona. There are sona of different dimensions and shapes which nevertheless follow the same geometric algorithm. In chapter 3 some exercises are presented where to some given series of monolinear sona other elements of the same type are asked to be found. The forth section has been resulting from workshops with mathematics-teacher students who experimented with the educational and scientific potential of sona. During those workshops while exploring sona it had been observed that among the participants there was such striking awareness that time perception ceased altogether. The made experiences suggest that by the discovery of the didactic and mathematical potential of this cultural heritage the young teachers can gain a high convidence not only in their capability to understand mathematics, but also in the great potential of the African culture as a whole. Formally the forth section is dealing with rules of composition of different drawings, with the systematic construction of monolinear patterns and last but not least with the generation of a specific class of lusona in different dimension, namely the so called "fleeing cock".

Chapters 5 and 6 give more detailed analysis of the construction of sona of the type "fleeing cock". Especially chapter 6 studies the route of the cock in terms of generating reflections. When the fundamental geometric space is represented by a rectangular cellular grid R[n,m], it can be shown that the route of a monolinear lusona brings upon a fundamental orientation of the elementary cells of the grid, which can be understood in terms of a modulo-4 counting of the cells through which the cock is passing. At the same time the grid can be transposed onto a *"magic modulo-8 rectangle"*. Gerdes shows that each fundamental grid R[n,m] possesses a natural Q-enumeration of its cells. Further every *monolinear, regular, plain mirror pattern* (such as the *"fleeing cock"* or the *"lion's stomach"*) induces a specific P-enumeration of R[n,m], which is given by the course of the monolinear drawing (the route of the cock through the grid). A proof is given that the natural Q-enumeration of the cellular grid equals the P-enumeration induced by the sona. The basic idea that will be derived from Gerdes' procedure is that movement of the body structures space and gives it local orientation. The pattern of local orientations is a temporalization of movement. That is, time occurs through a measure given to spatial motion, as has been said by Aristotle.

Those readers in the first world who are familiar with the theory of geometric algebras will immediately be reminded of the role of modulo 4 and modulo 8-periodicity in Clifford algebras $Cl_{p,q}$. The table of representations of Clifford algebras continues with a periodicity of 8 while dimensions p, q increase, and the division rings **R**, **C** and **H** (real, complex and quaternion numbers) are repeating with (p–q) mod 4 ≠ 1. As a matter of fact, Gerdes reduces a lusona (line in a cellular grid) to a set of elementary orientations given by the eight symmetries of the dihedral group $\mathbf{D}_4$. This can best be seen from figure 6-14 on page 282. Monolinear sona are decomposed into regular, cellular grids the cells of which are symmetry-valued. That is, the value of each cell is one definite symmetry of $\mathbf{D}_4$. It is well known that the orthonormal basis sets of Clifford algebras generate finite groups. Those are denoted "multivector groups", and the multivector group of the Clifford algebra $Cl_{2,0} \simeq Cl_{1,1}$ of the plane is the dihedral group $\mathbf{D}_4$. Yet, Gerdes does not use any of our results from geometric Clifford algebra to proof his theorems on monolinear mirror patterns. Rather, he carries out a didactic exploration starting with the most basic: that which can be observed by investigation of the sona in a precise, but almost entirely empirical way.

Chapter 7 investigates generation and enumeration of monolinear, regular mirror patterns that belong to a given dimension (r, s). Chapters 9 to 12 represent comparative studies of cultural elements from ancient Egypt, Mesopotamia, India and "other continents", strictly, ornaments with celtic origin, drawings from the Vanuatu-islands and monolinear patterns from North-American Indians are shown. Many of the beautiful patterns engraved on the bottom of scarabaei are identical with Tshokwe sona: the *"two birds in their nest"* or the *"bat with the folded up wings"* which is the same as the Egypt hieroglyph for *"plan"* or *"fundament"*. For some of the drawings from Mesopotamia, it can be shown that they do not originate from wickerwork. Most of the Indian patterns that were drawn at the tresholds of Tamil houses like the Pavitram-patterns or Brahma-mudi (Brahman's knot) or other drawings of the kolam type can be reconstructed in terms of the rules discovered with the Tshokwe sand drawings. Rules of composition for monolinear drawings seem to have been known in such distant regions of the world as ancient Britannia, the New Hebrids or Mexico. Some of the African patterns which are shown in the last chapter *"Return to Africa"*, which Baumann denoted as *"typical African"*, are indeed simply D_4-symmetric. They represent the orientation symmetry of the Euclidean plane or the coordinate-(anti)automorphism of the planar line-cross. This is best symbolized by decorated tunics from Senegal or Ethiopia and by Ghana's *"knot of a wise man"*.

Gerdes' book gives an exquisite survey of both, questions of Ethnomathematics and modes of its questioning. It does not only explain the generative principles of the Tshokwe Sona Geometry, but it even vividly represents to us how mathematics is recreated ever anew by human cultural action. The author is radical inasmuch as he does not use any pregiven concept or theorem of geometric algebra from the First world. But he adheres staunchly to the principles of cultural creation which requires that he goes back to the roots of cultural elements such as wickerwork, tattooes, sand drawing — and human action in general which, after all, bring forth the articulation of mathematical thought. Remaining true to the socio-historic process of manufacture, investigation and didactics, he thus promotes a precision of thought and a vividness of mathematical reasoning which can outdo many of our present day textbooks in advanced mathematics. Paulus Gerdes' text-monograph is an invaluable pedagogical resource for the Ethnoscience communities engaged with non-eurocentric, non-universal Mathematical Science, and for the cultural anthropology, ethnology, sociology and mathematics community. For those who are interested in the (pre)history of mathematics, for them who wish to "stimulate cultural awareness in mathematics teachers", for teachers, for those who wish to reconstruct lost symmetries, or those who want to make better hand bags, who are looking for missing figures in ornaments, for curriculum researchers, for politicians in mathematics education and basket weavers, for women, men and children Gerdes' book represents values beyond words.

The findings of Ethnomathematics can contribute significantly to modern social theory. By investigating the *Sona Geometry* it gives us a rather general example for a theory of a social practise where experience, symbolic and discursive forms go together unparted. I found out that what has been regarded as an old educational acquisition of stone age worship — the template of orientation ⊕ — operates as a principle of structuration in all those stories articulated by Sona sand-drawings. What strikes me most is the similarity between the Sona concepts of geometry and the newest approaches to quantum physics (Cantorian Geodesics and Fractal Space-Time; see: El Naschie, Rossler and Prigogine 1995). It sometimes seems to me that people without writing have a more appropriate image of reality than western physicists, and that physics, after all, should be transformed by social theory.

6 Concluding Comprehension

Since long symbolic order and discursive forms have been considered as institutions of signification. Anthony Giddens has also used the concepts of *discursive consciousness, practical consciousness* and last not least the *subconscious* in the sense of Freud, Erikson, Laing and Goffman. Following his argumentation line, we have gone a little further and introduced awareness as a bioenergetic agent which is capable to activate both the discursive and practical consciousness and body language. That is, we have to consider both cognitive awareness and body awareness as active parts of social structuration. In this way, we obtain a dynamic bioenergetic system involving cognitive awareness, practical consciousness and body awareness, and all three structure social space and temporalize social action. Time is a measure of movement which is essentially bound to orientation in social space. Actors activate those three components of awareness to different degrees and with different orientations and symmetries and symmetry-breakings respectively. Our experience of each other and our images of each actor's image of other actors experiences and images is regulated by bioenergetic social phenomena in temporalized space. Each actor's presence is physical, symbolic, discursive and bioenergetic at one time. Many combinations are possible. For example, an actor can be physically present, symbolically present, present in the conversation, bioenergetically absent in his own experience, but bioenergetically present in the others' experience. Also someone may be physically absent, but symbolically present, bioenergetically present in others and even influence the discursive form and so forth. Looking at social life in this way, social phenomenology and symbolic interaction are linked to bioenergetic processes of orientation, social action and temporalized social space. In any case the body appears as an institution of signification and what was denoted *interpersonal perception* or *interexperience* is structured in and structuring social space.

It may be thought that such a view is entirely incompatible with precise mathematical concepts. But it is not. In positing a theory exact concepts are even necessary to attain the highest possible level of clarity. We only have to abandon the idea of a deductive theory and develop cognitive frames not as total structures but in the awareness that concepts of space and time differ from culture to culture. Therefore, we emphasize the findings of that joint venture of mathematics and cultural anthropology: *ethnomathematics.*

Reconstructing Orientation and Space

- Second Reconstruction -

7 Prologue on Presence

There is a prevailing image about the nature of time which brings confusion into the sciences of both nature and man. This image is resulting itself from the difference between physics and sociology. In the great old greek treatises those two and philosophy went together. But history established an intervall. It is therefore that we have to reconstruct space-time. We must ponder over its history. Considering that — as Giddens drawing on Heidegger, pointed out — human subjects differ from material objects by their temporal character, we must understand the emergence of time perception, which means to comprehend inner experience (Mohr 1992, p. 198f). This requires in addition that we look into prehistory. Because it was then that a separation between inner and outer came into being. In a period of about 40000 years lenght until about 1000 years before Christ, the observer and the observed were one, and the main form of consciousness was participation. Men felt one with their environment and their absence as observers was a presence out there. Morris Berman (1984, p. 176 f.) denoted this time intervall as a period of *mythical consciousness.*

The emergence of time is bound to the discovery and the invention of various ideographic symbols. Time as a serial sequence of motion, that is, the experience of linear time, comes on simultaneously with the linear writing. This has been pointed out most beautifully by Vilém Flusser (1992) in his booklet *„Die Schrift".* There was less of time than there was presence, before time could appear. Time comes in through the absence. We become aware of time when our attention is drawn away from an object of observation or from some current activity. Time is a break to the awareness of presence which limits a duration. Time is brought on by the appearance of absence. The appearance of absence in its essence is the same as the intervall between the observer and the observed. When this intervall vanishes there is participation.

In Aristoteles' enigmatic definition of time its most important modality of appearance cannot be found at all: *>Time is the measure of movement in regard to the sooner and the later<.* Nothing, however, is said about presence; how come? Presence does not belong to time. Though any story may have a beginning and an end, such beginning can only be ascertained in the now, and such end is to be verified in the now too. Whatever has been, is or will be, must be established in the vivid presence (e. g.: Figal, 1992, p. 39).[3] The observer of an

[3] The denotation of "vivid presence" is also used by Schütz.

electoral campaign states that the campaign is beginning now, and after some time he says that it is ending now. Time emerges in presence, but the present is not of time. That this is a significant thought can be seen from Whiteheads statement: *"There is no essential reason why memory should not be raised to the vividness of the present fact; and then from the side of mind, what is the difference between the present and the past?"* What led Whitehead to that supposition is subtle: *"The distinction between memory and the present immediacy has a double bearing. On the one hand, it discloses that mind is not impartially aware of all those natural durations to which it is related by awareness. Its awareness shares in the passage of nature. We can imagine a being whose awareness is our own transient nature. Yet with this hypothesis we can also suppose that the vivid remembrance and the present fact are posited in awareness as in their temporal serial order" (Whitehead 1964, p. 67).* Being aware of the deep meaning of Aristoteles' definition of time and Whitehead's concept of extension, what is the significance of the first sentence of the following reflection by John Urry on Giddens' statement? *"The movement of individuals through time and space is to be grasped via the interpenetration of presence and absence, which results from the location of the human body and the changing means of its interchange with the wider society"* (Urry 1996, p. 381).

It can only mean that social change must be seen in terms of an interpenetration of time as measure and that which is beyond time. If we want to reconstruct time, we must decode and comprehend the meaning of certain historic and prehistoric events in terms of the present. All that has been is contained in symbols and things, which are here in the now, physically, geologically, organically, biochemically and informationally. The situation is somewhat complex because the *"object"* under investigation is the emergence of time or put more puzzling: *the constitution of time consciousness by the time.* What appears by the time is time itself, and the whole process of the organization of time has to be here now. Otherwise we could not reconstruct it. But (**Question 1**): what does it mean to explain the constitution of time consciousness by the time? Let's put the words apart! By a constitution we can only signify the whole of society as Giddens has done. By *time consciousness* we refer to both the individual self and society. Finally, by a constitution of time consciousness by the time we mean a constitution of time in history. This brings in some paradoxes.

(i) the constitution of time *is* history

(ii) words "by the time" are lent from physics (natural history) and refer to linear serial time

(iii) history is a myth of civilized thought (Lévi-Strauss)

(iv) there can be no natural history before there is history (Flusser).

All we can say for sure is that **Q1** reflects on the separation between physics and sociology. Further we can postulate that, whether time is in experience or not, time is constituted at all or not, is moving by the time or is not moving: human actors are moving in space, anyway. Therefore I would like to formulate the following primary statement:

There is movement without time.

This should lead to the observation that there is movement which has no temporal measure, because — as Aristotle and others have stated precisely — time is measure given to movement. It is measure that turns movement into motion. *"There is no sharp distinction either between memory and the present immediacy or between the present immediacy and anticipation. The present is a wavering breath of boundary between the two extremes. "The fundamental distinction to remember is that immediacy for sense awareness is not the same as instantaneousness for nature* (Whitehead 1964, p. 69 f.). *"The process of nature can also be termed the passage of nature. I definitely refrain at this stage from using the word 'time', since the measurable time of science and of civilized life generally merely exhibits some aspects of the more fundamental fact of the passage of nature.* (p. 54) *Thus not only is the passage of nature an essential character of nature in its role of the terminus of sense-awareness, but it is also essential for sense-awareness in itself. It is this truth which makes time appear to extend beyond nature. But what extends beyond nature to mind is not the serial and measurable time, but the quality of passage itself which is in no way measurable except so far as it occurs in nature"* (p. 55). Accordingly, the experience of body awareness is not necessarily bound to time experience. There is body awareness beyond the measurable time. Yet it involves passage. Since *"There is no such thing as nature at an instant posited by sense-awareness"* (Whitehead 1964, p. 57).

It is because of all that has been said until now that we first reconstruct space, before we go into this matter of time. The organization of space will turn out fundamental for the constitution of time. Before the occurrence of measurable space there is non-measurable space. Non-measurable space involves concepts of orientation and symmetry. Thus, at the origin of human society, there can be found an algebraic concept of space. This concept is algebraic not only for us, but also it was algebraic for them. Algebra means to set right (German: *'einrenken'*), to put broken parts together.

8 Non-Temporalized Space

To reconstruct the space of human orientation we first got to understand the institutional order of the non-historic mind. The institutional order of signification, according to modern theory, is given by discursive forms and symbolic order. The discursive consciousness of actors is reacting to lived differences between fact and ideation. Large domains of signification, however, are not accessible to the discursive knowledgeability. A great deal of knowledge in stock („*Wissensvorrat*" after Schütz), much of the *habitus form* (Bourdieu), much of the *common knowledge* incorporated during encounters (Giddens) is not directly accessible to human consciousness. Part of it is hidden within the routines of social life as *practical consciousness*, part of it is subconscious in the sense of psychoanalytic theory. Taking into consideration the typologies of forms of interaction developed by Goffman, Ronald Laing and Giddens, it is clear that

(i) in a prelingual society, those discursive forms based on verbal communication are reduced, and

(ii) those discursive forms based on perception differences between images the actors have: of themselves, of others and of images others have of . . . and so forth (Buber's ghost) are reduced, but

(iii) nevertheless, just as in nowaday societies, so in non-historic communities the order of interactions may be seen as anchored in the most general attributes of the human body (Sacks & Schegloff 1974, Giddens 1984, p. 77).

It is surprising that Giddens discussing the active organization of space (*"spacing"*) following Goffman (and even Erikson) ponders over the meaning of front- and backside in communication, but does not realize the importance of orientation of space as a whole. Neither in social theory, nor in modern mathematical physics have we become aware of the significance of orientation. After all, the institutional order of signification is resulting from an incorporation and socialization of spatial orientation. When the experience of movement beyond time is linked to the habitual form of orientation, there appears time as inner experience. It is therefore that the so-called operational structures of space, time and logical thought were disclosed as *psychogenetic structures*. So-called inner experience is derived from the bioenergetic action of a neuronal filter which is responsible for orientation in outer space. But this model, it seems to me, does only hold true as long as there is a separation (separatio et distinctio) between inner and outer experience. Beyond the separation, neither do individuals move in space outside, nor do imaginations and ideas move in time inside. Time and space are abstractions from more concrete elements of nature, namely, from events (Whitehead). As a matter of fact, it is not necessary to think about space and time as given conditions of perception. Rather both concepts are bound to the reconstruction of society.

9 The Concept of Orientation

To understand space and time it is not enough to follow the traditional argumentation line of mathematical physics. But we have to understand that they belong to a social manifold of discursive forms having itself an extension in social time-space. Discursive forms dealing with space and time differ from people to people. Often they are connected with story telling and give articulation to social phenomena without involving measurable or continuous space in the western sense. Nevertheless, they represent exact concepts in a mathematical and linguistic fashion. This means that we must first reconstruct the concepts of non-historic, historic and ethnic communities and comprehend how those contribute to a totality of symbolic order which includes our present concept of space-time as only one of its forms. Even the concept of mathematical physics which is believed to be highly advanced can be understood in all its consequences only if the other, seemingly more primitive concepts, are taken into consideration.

It would not be clever to entirely abandon mathematical language in situations where the ethnic conception already takes an exact abstract shape. But as was said in the fundamental statement on *"Refinement of Social Theory"* (division 5), we should not impose western standards on peoples without writing but instead use the findings of ethnomathematics. Ethnomathematicians in many parts of the world have explored very old cultural techniques such as beadwork, braids, wickerwork, texture, decorated carved work and sand-drawings to discover and study *"hidden or frozen"* mathematical knowledge (Gerdes) without destabilizing the existing cultural forms. In this way our present procedure of social theory becomes rooted in that joint venture of mathematics and cultural anthropology which was begun about twenty years ago in Africa (Angola, Mozambique) and America (Brazil) and is connected with the names of D'Ambrosio, Gerdes, the Aschers, Ferreira, Bishop and others.[4] The ideogram of orientation ⊕ is known in ethnomathematics, but is a paleolithic concept. Bauman (1929, 61) claims that symbols (with a dihedral symmetry) such as

Figure 1: African ornaments (Prussin, p. 91 and Baumann, p. 61)

[4] Detailed information can be found under http://web.nmsu.edu/~pscott/isgem122.htm

are *"typical African"*. Motives like that still appear on clothes of the ethnic group of the Haussa of Nigeria. But the original symbol of ⊕ is a stone age cave ideogram. It has been photographed, drawn and painted on the expeditions of Frobenius (*"Hadschra Maktube"*) in some caves close to the Erg mountains in North Africa (Frobenius 1939). Also Marie E.P. König and her daughter have taken many pictures during their investigations of the caves in the Ile de France. The ideogram ⊕ represents one of a few symbols which arose from cultural activities so old that it seems that no actor can show us or give us a hint how it was constituted. It appears that although the symbol of the quartered circle is well known in ethnomathematics, the circumstances of its origination cannot be reconstructed by thawing out or waking up knowledge frozen in handicraft. Yet there is enough archeological material so that we can try to reconstruct the sociological origin of that old institution of orientation without having any stone age-informants.[5] We have to discover the meaning of that old symbol before we go deeper into spaces disclosed by ethnomathematics because it can be used there as a significant element of structuration. And most possibly, as we shall see in a while, a symbol of this or some very similar form has always been used throughout prehistory and history as an ordering tool to give space and time a genuine structure. Consider the original symbol of orientation in the form of

⊞ + × ⊕ ⊗

Figure 2: Form of Ideograms from Europa and Africa

Those appear in our paleolithic past and are well established by the Moustérien sometime about 40000 years ago. Some scientists have realized that it may have been easier to establish one definite direction in space by sun observation, as is represented by

| I H ⊘ Δ ⊟

Figure 3: Form of Ideograms showing one distinguished direction

and that it took an additional cultural effort to draw the orthogonal line cross ⊕ that can be found in so many European and African caves (König 1981). In astronomic terms this fact may be regarded as obvious. Sociologically, however, it is not at all trivial. Face to face

---

[5] I have tried to reconstruct some of those non-historic events in the IHS research memorandum Nr. 313 (Schmeikal 1993).

interaction symbolized by " | " refers to a distinguished direction in social space. But the body has an *approximate* lateral *symmetry* in addition. So when two of us are standing or walking side by side this signifies a state of coordination and mutual dependence. As most of us know, it makes a difference if you walk alone or with another. It makes a psychological and bioenergetic difference. Even the manner of walking is changing when two of us go together as compared to when they walk alone. This is most impressively realized by a Tshokwe tracing called *"united couple"*

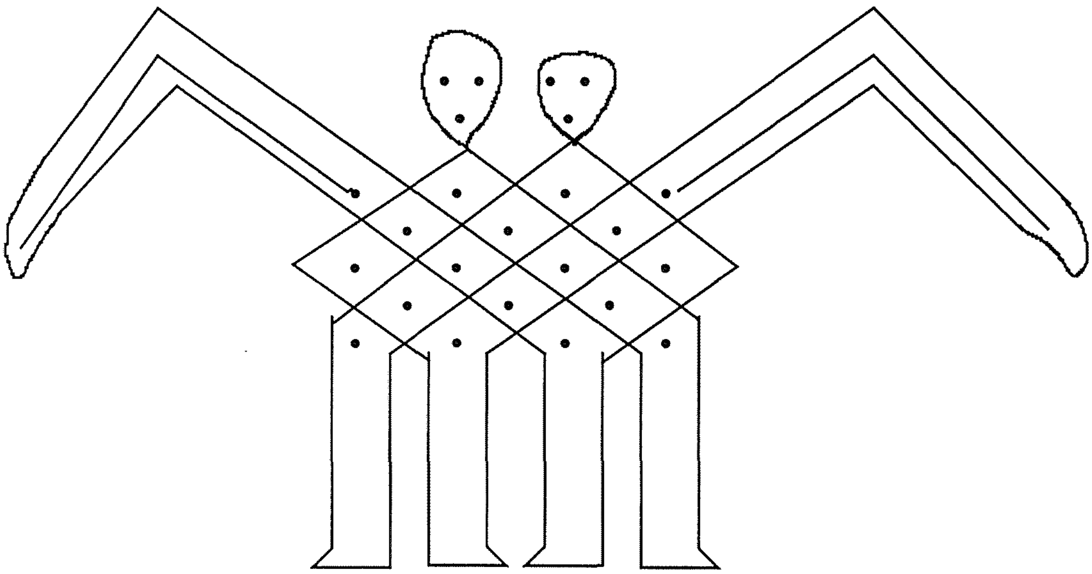


Figure 4: The monolinear Tshokwe sand-drawing *"united couple"* (Gerdes, p. 158)

What is so peculiar about this lusona (lu-sona denontes *sing.*) is its monolinearity. Being aware of Fontinha's work (1983), Gerdes (1997, p. 36) has pointed out that monolinearity can represent a cultural value Fontinha has collected and published 141 drawings of 10 different types, and 61 of them were monolinear, that is, drawn in one line. They were drawn by 8 different masters or experts (the *akwa kuta sona*): one of them, named *Samesa,* drew 13 sona out of which 12 were monolinear, another *Sacapacata* drew 19 out of which 16 were monolinear, *Chizainga* made 18 with 13 monolinear ones, and *Saitumbo* 15 out of which 10 were monolinear To be able to draw a lusona without interruption requires a high degree of attention. In high attention the feeling of time disappears.[6] This is a hint for social theory:

[6] *"In 'workshops' mit Mathematiklehrerstudenten in Mosambik haben wir mit dem erzieherischen und wissenschaftlichen Potential der Sona-Tradition experimentiert. Die Teilnahme an diesem Seminar war frei, und im allgemeinen war es schwierig, die Sitzungen innerhalb der geplanten zwei Stunden zu beenden, da die Teilnehmer ihr 'Zeitgefühl verloren' hatten".* (Gerdes 1997, p. 22)

Presence in active awareness promotes unfragmented space.

When the akwa kuta sona makes the experience that he cannot draw the sona in one continuous movement, he becomes aware of a disability. No longer he regards himself as a master. Looking at the *"united couple"* with this in mind, we understand that the two are united by a single line in the active presence of the *akwa kuta sona* who carries out the tracing. The space-connecting line symbolizes the presence of interaction, and this interaction is organized along the axis coordinating the lefthand- and righthand-side. The *monolinear pattern* is seen as originating from the active presence of the drawing expert. It symbolizes *vivid presence* (drawing on Schütz) in social space: Its form is the united couple.

Figure 4 gives us an image of social interaction along what we call the *x-coordinate* (left-right). Let us add to this a *face to face-interaction coordinated along the y-axis* as was described in the first report *(Refining Social Theory)*: Suppose we are standing face to face. While my body — the body of ego — moves towards you, the body of alter, I experience that I move foreward and approach you and you experience that I move foreward and approach you. Both experience that we approach each other. Also you and I know or/and experience that my right hand is on the side of your left hand and my left hand is on the side of your right hand. Both of us share the experience — if only unaware of it — that what is above is above both our heads and what is below is below both of our feet. Most of the physical space infront of you — is behind my back and most of what is infront of me is behind your back. Yet standing face to face, while I am aware of you — gestures, speech, eye movements and body language — **most of what is in front of me is you** and is therefore not behind your back. It is only when I loose attention that what is in front of my face falls into the unmeasured space behind your back. And only when our attention is reduced, in the instance, we may be unaware of what is above, yet we say it is clear there is an above. Also we may not know what is below our toes and ankles. Yet we say we are sure there must be something below our toes and ankles. All this is influenced by practical consciousness and by the subconscious, and that all this is as it is, is as it is wherever we go and in whichever Kulturkreis, culture or society we are moving. The most basic operations of orientation form an invariant core of experience. In physical space those appear in the disguise of algebraic invariants. In mathematics, they form a total structure (of algebraic group theory). However, in social space the experience of those invariants is temporalized through the structuration of the copresence of actors. In our presence not all of physical space is present all the time. Social space is structurated while in the actors' copresence social meaning is constituted by a synergetic arranging of absences and presences. That is, by moving eyes, face, arms, legs and the whole body, the attention af actors can be manipulated such that the actors may appear more or less present or even vanish in social space. Temporalization of social structure implies decomposition of social space into bundles of absences and presences. A figure like the following

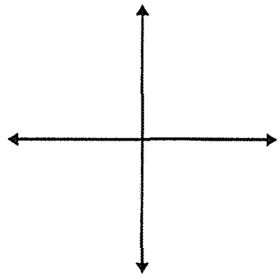


Figure 5: Line cross or coordinate system — idea of an ideal space

represents the *idea of an ideal space* (**R**^{2}) which is *no total concept* in social reality. In social space, presence is not a topological invariant, but it fluctuates. Social interaction temporalizes such a "coordinate system": coordinates are uncertain and cannot be considered as having infinite lenght all the time and under all social circumstances. But under microsociological conditions they are contracted and dilated and in permanent motion. Space, for a living being, is not objectively here, but it is always decomposing and reconstructed. It has no objective reality. Only in scientific ideation has space a reality as a three- or four-dimensional continuous space. In social space we rather have to locate a moving fractional space with varying features of symmetry, compactness and measurability. But also this is only a *medieval intellectual model*, to use Whiteheads metaphor. It does not come very close to reality. Even physical space cannot be conceived in an absolute manner and independent of social reality. It is only a cognitive concept, a symbolic form. When views and theories change, continuous 3-space, even in physics, may turn into discontinuos space of variable, non-integer dimension.[7] Now, does this mean that the cognitive concepts of orientation and physical space have no practical reality at all? Is the line-cross, Jung's Mandala, the psychogenetic structure of orientation, the morphogenetic structure of experience — or in whichever way we speak about it — is our *"coordinate system"* only an ideal idea without any practical relevance?

Not at all! It represents a part of practical consciousness. Under healthy conditions no person has difficulties to go straight, turn left, turn right, move up or down with a lift and go wherever she wants. The template of orientation is bioenergetically activated most of the time. Only when we sleep may it be in an off state. That is, the ground structure of orientation is basic for the action of practical consciousness. Orientation is needed in the activation of bioenergy or body awareness. We articulate our *"orientation in social space"* through body language. Thus, it is basic for both body awareness and practical consciousness. As I have worked out

[7] It is now thought about scale relativity and fractal space-time (Nottale 1995).

in some previous works (1993a, b) the morphogenetic structure of orientation can be brought in a one-to-one correspondence with the operational structures of thought (Piaget 1957, 1981). Therefore it turns out as a most interesting enterprise to study if and how it can be continued into the symbolic forms of cognition. That would mean that orientation is active in all the institutions of signification, body, practical consciousness, subconscious and discursive forms. This, in addition, legitimates the supposition that most of what we are doing must act in such a way that it reconstructs and stabilizes this ground pattern of experience.

There arises the question — for the first time in these reconstructions — why the concepts of physics seem to be so much different from those of the social sciences, so much more reliable and stable. Space seems to be three-dimensional to most of us and so forth. The reason is not that physical concepts have an objective reality which social space does not have. It is rather the other way around. Physics is based on concepts which are reconstructed by almost each of us, all the time and since such a long time that almost each of us is entirely unconscious of their reality (reality in the old greek sense of ***wirk***-lichkeit).

> We can be sure,
> we believe, that,
> suppose we die in the midst of a dialogue,
> face to face with our partner,
> and fall down to the ground,
> the space behind our partner's back, where the attention got lost, that is,
> the physical space behind his back that was formerly infront of our eyes:
> is still there.
> We believe that, and, in a way, it is true.
> In which way, we may ask,
> is space there, in case we all fall down?

Or does it amount to the question of which form of consciousness or which form of intelligence is active once we are all gone and vanished? Is there a form of awareness, or consciousness or some intelligence which has a physical, material institution, but no practical and no symbolic? Something that cannot be talked about since it is beyond all possible forms of discurs? The answer to this is a mere idea! A form in cognition. Or does it have an extension in body awareness?

Wherever and whenever the attention decomposes, it seems, whenever body language becomes rigid and fades away, where social life is destructed and death comes ahead, social space collapses into physical space, and only physical space survives.

We believe that, but it is an illusion.

It is a remarkable fact that those of us who have the objective view and therefore regard themselves and others as objects with objective properties, affairs and fate, also believe in the objective fact of physical space. But those *"phenomenologists"* who experience themselves and others as greatly alive, are not ready to separate space from the living.

To the objective ones everything appears to be an object. For the living everything lives: "The relation of experience to behaviour is not that of inner to outer. My experience is not inside my head. My experience of this room is out there in the room. To say that my experience is intra-psychic is to presuppose that there is a psyche that my experience is in. My psyche is my experience, my experience is my psyche" (Laing 1975, p. 18f).

Wherever and whenever individuals come together in a specific context, they measure out the space between them, and they do it consciously, automatically and subconsciously. They activate their images of others and their images of the images the others have of them, they estimate the multiple differences between those metaimages and the images the others have of themselves. Depending on their intellectual capacities they consider a comparative route within that space of interpersonal experience, but not only that. To answer that question which everyone is confronting, namely: *"what is going on"?*[8] they must estimate the multiple constellations in emotional space which is a matter of bioenergetics. An emotional constellation is nothing other than a regionalization of bodies, theirs and others. Our body awareness is regionalized according to and in synergy with the constellations of emotionally regionalized bodies surrounding us. If someone whose presence is unpleasant to us, whose images are disagreeable or whose behaviour appears troublesome, is standing close to any one side of us, that side will react bioenergetically. It will be activated in aggression or it will be blocked in order to promote our avoiding, our stepping aside and our turning away. Most of our physical operations in physical space are provided by practical consciousness and by our subconscious. They do not require a lot of thinking. So let us ask once more: What is going on? What is our intelligence doing? How is the awareness moving? In what way is it active in which parts of the institution of signification? The intelligence of awareness works in interpersonal perception space[9] and in interexperience. It is also operating in and measuring out the emotional space, physical space and in the body. Awareness brings on vivid interaction in all the layers of consciousness: cognition, practical consciousness,

---

[8] This question is dealt with by Giddens (1984, p. 87) in his analysis of *positioning*. It has also been posed in the first Tarner lectures on Nature: "In the first place there is posited for us a general fact: namely, something is going on; there is an occurrence for definition". (Whitehead 1964, p. 49)

[9] This I prefer to denote "interpersonal cognitive space" because it is a matter of cognition, construction and intellect (I refer to Laing's IPM). It is not identical with interexperience.

subconscious and body, and it is anchored in body awareness, physical awareness! Why are we believing that the physical stratum of existence is excluded from intelligence? We have understood that social space, interactive space, bioenergetic space, cognitive space are constantly produced and reproduced in vigilant interaction. Why do we regard physical space as given? Why are we not asking how physical space is reproduced? Matter and space are one thing. Particle creation is space creation. Which form of intelligence is operating in so called anorganic matter? In "empty space", in the *"vacuum"*? There is neither good reason nor a good proof to hold it for certain that the awareness is entirely cancelled out in cases where the conscious and the subconscious mind, the practical and the body awareness cease to exist. When everything ceases to *exist*, that which is *insisting* is still insisting. This *insistence* of intelligence (which is not to be confused with the intellect) is the reason why we believe that there is space independent of consciousness which amounts to the belief that there must be still something left of space when every form of consciousness is cancelled out. That which is left, we believe, is physical space: objective space.

However, this is not what is left. What is left, is the interactive material basis of material life, the music of matter or as Whitehead put it, that factor in sense perception which is not thought but awareness, and also nature is not thought. The multidimensional, variable intervalls in the vivid presence of matter — the space where our experience is, in case we are here — is what is left. We do not know anything about the various forms of intelligence — structural layers of awareness — brought forth by and in matter and independent of the human institution of symbolic forms, thought and discursive consciousness. Tracing back the evolution of intelligence, it does not halt where thought decomposes, and going the other direction: it does not begin with the inset of the constitution of thought.[10] There are ethnic communities and civilized peoples where that is the prevailing view. In such a view human life and physical matter are not disconnected. The object is an object, indeed, but it has a life of its own. Refering to such a view, consider the quartered circle as a vivid structure subject to constant reconstruction :

[10] „Today, some people still set out. But perhaps the majority find themselves forced out of the 'normal' world by being placed in an untenable position in it. They have no *orientation** in the geography of inner space and time, and are likely to get lost very quickly without a guide. * Orientation means to know where the orient is. For inner space, to know the east, the origin of source of our experience. (Laing 1975, p. 138)

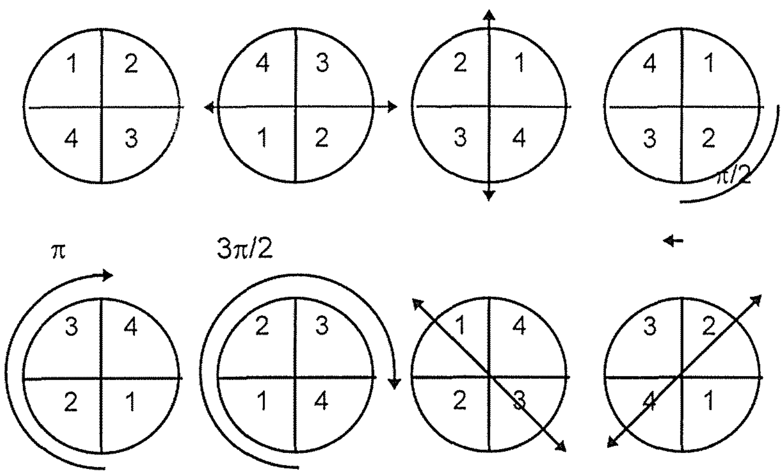


Figure 6: The 8 fundamental symmetries of plane orientation, axis of rotation are indicated by double arrows

It is often symbolized by the sun-wheel or eightfold path diagram

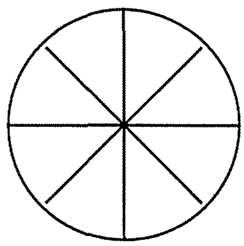


Figure 7: Sun-wheel or eightfold path diagram

This is not only at the basis of algebra and genetic structuralism (Piaget), but it represents a bifurcation point in the development of scripts. Not only the European alphabets, but also the proto-Elamic script and other archaic writings are organized around the concept of orientation.

The ancient city planning of *"Roma Quadrata"* or India's great *"silpa sastra"* as well as numerous groundplots of cities all over the world divide their societies into quarters of power and dignity. The sunwheel is a symbol of the structuration of power. Its sociogeographic meaning is fixed in words such as *"Landesviertel"*, *"Stadtviertel"* and *"headquarters"*. Its ultimate expression can be found in the *Quaternions of the Holy Roman Empire of German*

Nation.[11] There exist 4 dukes, 4 margraves, 4 landgraves, 4 burgraves, 4 graves, 4 barons, 4 knights, 4 cities, 4 villages, 4 peasants (builders), and the circle closes by denoting Köln, Regensburg, Konstanz and Salzburg the "4 peasants". The first elaborate *"social theories"* of quaternions can be found in writings of the 15th century, namely, in Felix Hemmerleins *"De nobilitate et rusticitate dialogus"*[12] and in Peter von Andlaus' *„De imperio Romano"* (1456) (in: Korth 1888, p. 118f.). The names of some of those medieval cities that have been built in accordance with the mytho-topology of *Roma Quadrata* are Worms, Köln, Münster, Florenz, Wels, Bern, Freiburg im Breisgau, Horn in der Lippe, Neubrandenburg, Villingen in der Baar, Lemgo Alt- und Neustadt, Rottweil, Wildungen, Kalkar, Brilon in Westfalen, Leipzig, Aachen and the empirial centre of Bamberg. But clearly the roots have a deeper location. The famous castles of the Vikings: Trelleborg on Seeland, Aggersborg am Limfjord, Odense auf Fünen, Fyrkat in Jütland. The political partition of Ireland in the middle ages, the "Landesviertel" of Karolingian Germany are attempts to anchor the infinity of imaginary coordinates within the real world and to connect it with power. Thus the morphogenetic structure of geometry as an image in ideation serves as a ground template for the establishment of power and social hierarchies. We have located the origin of this "powerful symbol" ⊕ in paleolitic times, and it is not by fortune that Müller points out: *„Das Bild der viergeteilten Stadt stammt aus tieferen Schichten, als sie den Theorien 'von Cäsar bis auf Karl den Großen' zur Verfügung stehen".* Müller quotes Gerber (1952, p. 463), who supposes *"Überprüft man die verschiedenen Quatuorvirate auf ihre örtliche Lage, so its festzustellen, daß sie in der überwiegenden Mehrzahl an den Grenzen in den vier Himmelsrichtungen gelegen sind, was die bewußte Absicht vermuten läßt, die Lehnsherrlichkeit des Reichs möglichst weit ausgedehnt erscheinen zu lassen".*

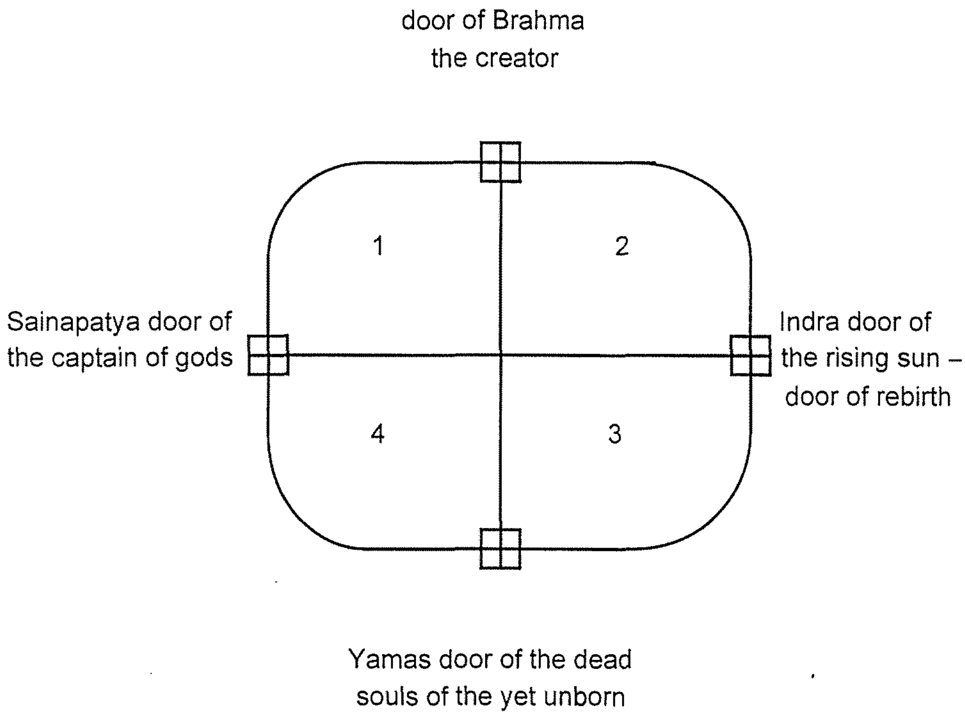
In India there is a collection of treatises on the principles of architecture called "silpa sastra" [13] (Dutt 1925) deeply rooted in paleo- and neolithic mythological thought. In the third reconstruction I would like to show the reader how a reversion of time can be conceived in such a system of thought. For an architect of ancient India to be instructed in the rudiments of town-planning by the silpa sastra was of the utmost importance. Composed by a mythical ancestor of architecture, a legendary sage who was familiar with the laws of cosmic construction, this knowledge goes back to Brahma himself, creator of the Universe. The groundplots of both Roman and Indian cities begin with the conception of the four main roads of the world which indicate the four quarters of the heavens. From the east to the west there goes the >*king of roads*< 'rajapatha', from south to north there leads the >*broad street*< 'mahakalapatha'. The >*path of favourable fortune*< 'mangala-vithi' surrounds the whole settlement. The most simple exposition of the silpa sastra concept is the 'dandaka' (=

[11] This kind of quaternion theory is not to be confused with the mathematical theory of quaternions. Yet, it is to be noticed that the quaternions of mathematics have a definite connexion with the algebraic properties of the quartered circle.

[12] The Text of the canon by Hemmerlein has been printed around 1500 and seems to be one of the protocols of the general assembly of the *Gesamtverein der Deutschen Geschichts- und Altertumsvereine zu Sigmaringen* 1891. It is mentioned in Müller (1961) on pages 98 and 247.

[13] silpa = handicraft, sastra = science

commemorative of a bar). The dandaka settlement has 4 main doors and 4 small gates placed in the corners. The following figure shows the groundplot of such a holy settlement.



In mythology time takes a cyclic course. To understand this consider creation in the Hinduist, Buddhist or Tantric images. Brahma gives an impuls to the wheel of life. He creates the world. His creation is not in time, but is permanent, and time is one of those creations. Creation, being itself beyond time, is pushing time foreward. Thinking in terms of linear progression, Brahma first creates a universe of gods necessary for the wheel of life to move on, Indra, Yamas and others and their captain god Sainapatya. Acting within the souls, their foregone emotions (in fact only 'clinging'), thoughts, words and deeds ('kamma' or 'sankara' within the dependent origination 'paticca-samuppada' in Buddhism), the psychic and mental formations are forced to reincarnate. Thus there occurs a transition from the region of dead psyches to incarnated life. We have now gone on the path of favourable fortune from Brahma to Sainapatya and from there to Yamas and further to Indra. This represents one temporal cycle of reincarnation. The revived soul lives on earth or in some other department of the universe and its ultimate aim is to become one with Brahma and to be liberated from the wheel of life. This step closes the circle.

In order to unfold the mathematical idea of this concept of time-space we have to realize that the above template of orientation has a symmetry or a group of symmetries of congruences, namely the dihedral group $\mathbf{D}_4$. If we use the enumeration of quarters as in figure 6, the 8 symmetry operations (briefly 'symmetries') are represented by 8 permutations of 4 elements. Those can be generated by two generating permutations, namely

[i] $C_2' = \Gamma_1 = (12)(34)$ [ii] $\sigma_d' = \sigma_1 = (1)(24)(3)$

Each permutation represents a definit enumeration of the quarters. Γ_1 for example represents the enumeration 2-1-4-3 and third subfigure in figure 6, σ_1 corresponds to the enumeration 1-4-3-2 and seventh subfigure. By forming products the whole group of 8 symmetries can be generated. Another more enlightening representation uses the instrument of Clifford algebra. In particular $\mathbf{D}_4$ can also be generated in the Clifford algebras $Cl_{2,0} \simeq Cl_{1,1}$ and $Cl_{3,0}$, namely by the unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$ themselves. That is, we have an algebraic correspondence between the Schönfließ symbols (Schmeikal 1996) and the unit vectors

[i'] $C_2' \simeq \mathbf{e}_1$ [ii'] $\sigma_d' \simeq \mathbf{e}_2$

which can themselves serve as generators of the dihedral symmetry of orientation $\mathbf{D}_4$. In the Clifford algebras $Cl_{2,0}$ and $Cl_{3,0}$ the product $\mathbf{e}_{12} = \mathbf{e}_1\mathbf{e}_2$ corresponds with the imaginary unit $i = \sqrt{-1}$. In the Pauli algebra $Cl_{3,0}$ there exist additional generators and dihedral symmetries corresponding with the other planes spanned by units $\{\mathbf{e}_1, \mathbf{e}_3\}$ and $\{\mathbf{e}_2, \mathbf{e}_3\}$. The bivectors $\mathbf{e}_{12}$, $\mathbf{e}_{13}$, $\mathbf{e}_{23}$ are the well known *quaternions* i, j, k of mathematics, the first *'hypercomplex'* numbers that have been discovered by William Hamilton (1843) and put into its proper place by William Kingdon Clifford. Any bivector squared gives minus unity. Most important is the double-group representation in the spin group **SU**(2) of the Pauli algebra $Cl_{3,0}$. In this algebra the dihedral group can be generated by the minimal set

[i''] $\sigma = (1/\sqrt{2})(\mathbf{e}_{13} - \mathbf{e}_{23})$ [ii''] $S_4 = (1/\sqrt{2})(1 + \mathbf{e}_{12})$

where the units $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ can be represented by the Pauli matrices σ_1, σ_2 and σ_3. Both S_4 and $\mathbf{e}_{12}$ can be taken as representative operators of a period-4 rotation. Powers of S_4 and respectively $\mathbf{e}_{12}$ represent walks on the >*path of favourable fortune*< 'mangala-vithi', that is, walks in time More details about Clifford algebra representations of orientation can be found in the appendix

10 The Space of Tshokwe Sand Drawings

The structurating pattern of orientation is not only found in non-historic communities. But it is also an implicit organizer in any Tshokwe sand drawing. We have selected the sona of the *'fleeing cock'* to demonstrate the meaning of archaic orientation in sand-drawing. Consider the *'fleeing cock'* in a regular grid R[5,6]:

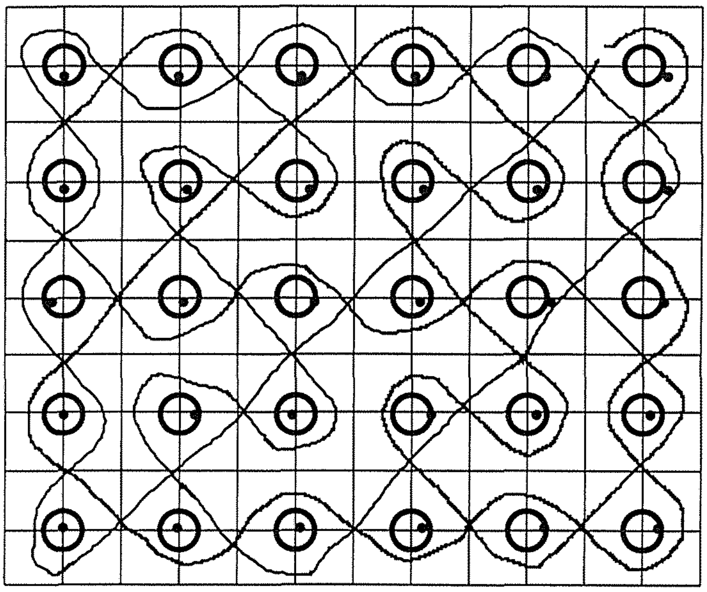


Figure 9: Lusona 'Fleeing Cock'

The route taken by the fleeing cock can be enumerated such that each cell in the rectangular grid obtains a definite number:

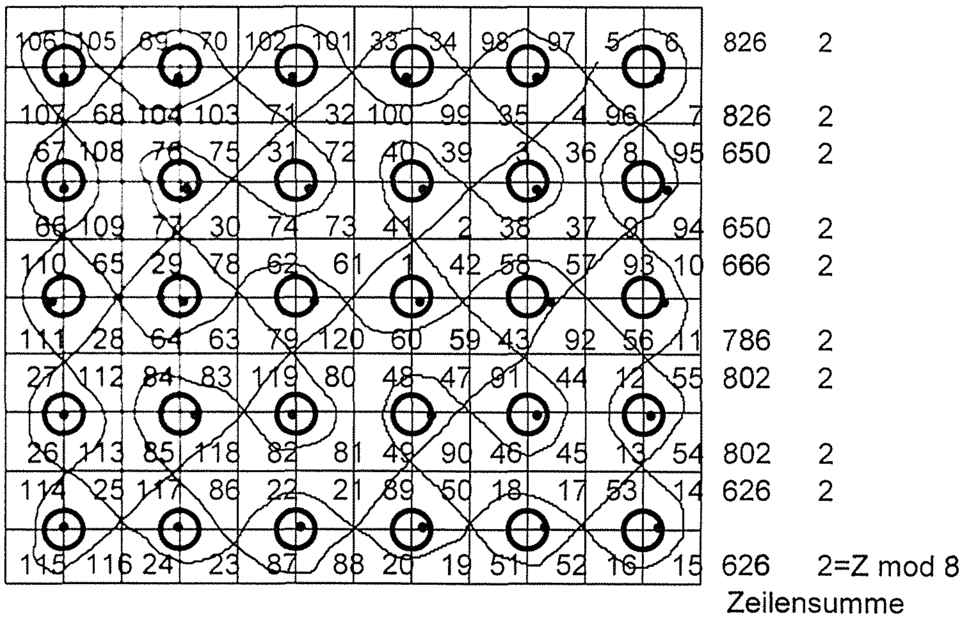


Figure 10: Natural enumeration of the cock's route

Gerdes (1997, p. 280) has discovered that this pattern essentially represents a magic array modulo 8. For any row, sums taken modulo eight give a remainder of 2. If we replace the natural enumeration by a modulo-4 enumeration, we obtain the following pattern:

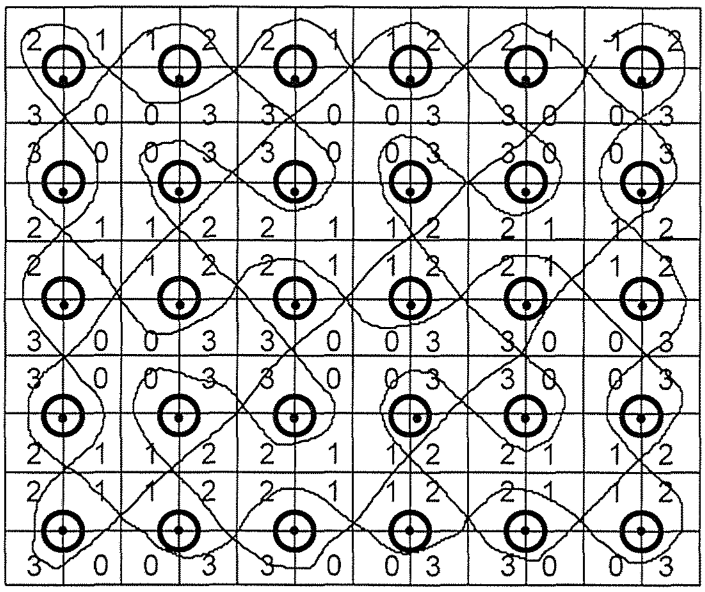


Figure 11: Orientation structuration of lusona 'fleeing cock' brought forth by a modulo-4 enumeration

Something important can be learned from that figure, namely, ***movement structures space by distributing local orientation.*** Thus, space is deconstructed or decomposed into a (here 2-dimensional) pattern of local dihedral orientations. The orientations of 'fleeing cock' are such that each cell obtains a definite orientation. If a cellular array at any given locus is running clockwise, its nearest neighbours situated either in the same row or in the same column are oriented counter-clockwise.

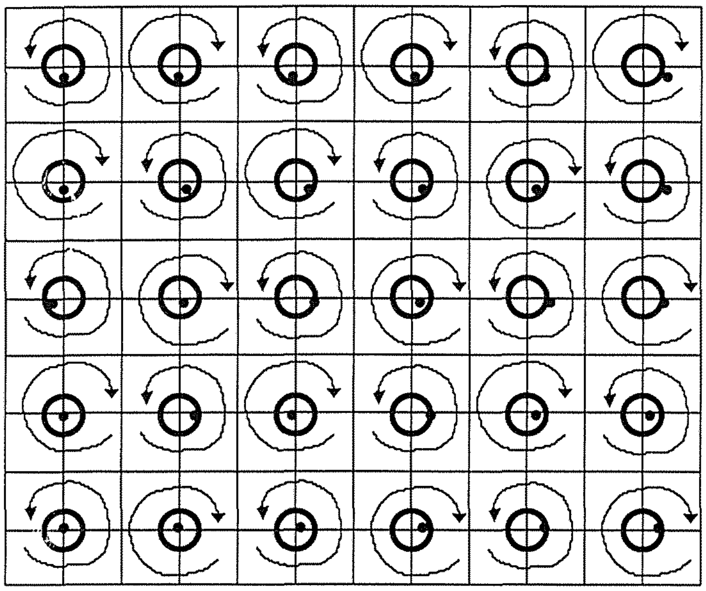


Figure 12: Convective patterns of local orientation in lusona 'fleeing cock'

This reminds us of some of those harmonious patterns of convective flow in fluid dynamics. Assigning to each centre of orientation a unit energy of angular momentum, we count 15 negative and 15 positive contributions which sum up to zero. This is probably the most important feature of such monolinear patterns (there exist infinitely many of them) that they coordinate space and movement in such a way that the whole event, in a way, sums up to zero. There is no remainder of angular energy, no remaining drift or direction is indicated. All circles are closed so to say and nothing is left. Perhaps this amounts to a mystery of awareness. Space which is connected in sense awareness often represents some sort of zero sum game. It would be interesting to study the type of orientation pattern that would be induced in the same grid of dimension 5x6 by some linear writing with return saccades:

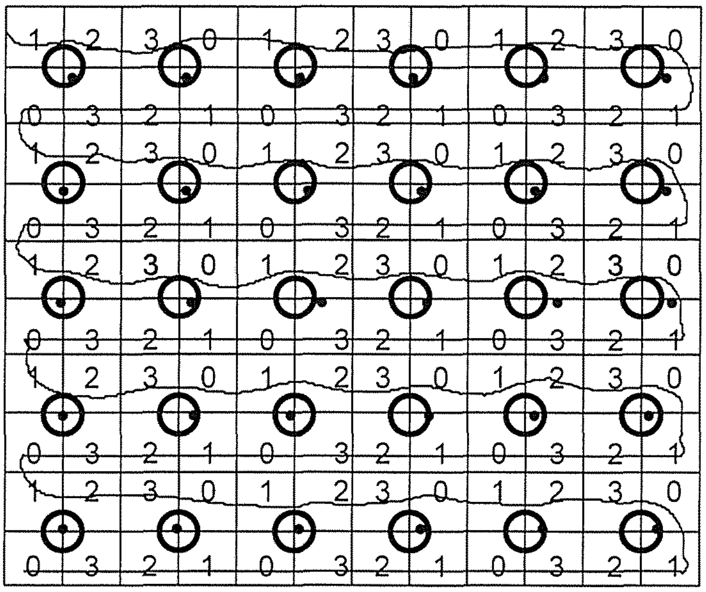


Figure 13: Structuration of the grid R[5,6] by linear writing

Note, all local orientations are now running clockwise (Figure 13).

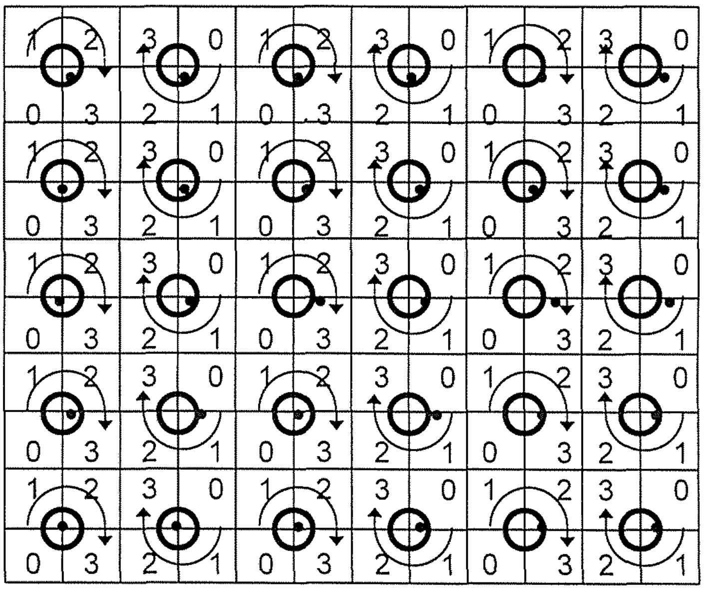


Figure 14: Structuration of local orientation by linear writing

Whereas decomposition of local orientation in the lusona is such that a synergetic pattern of motion analogous to heat convection or fluid dynamics appears, by a linear writing procedure a friction pattern is brought about. The energy of angular momenta no longer sums up to zero, but instead gives 30 units. The energy vectors block each other up. But it is possible to draw another monolinear lusona which integrates rows:

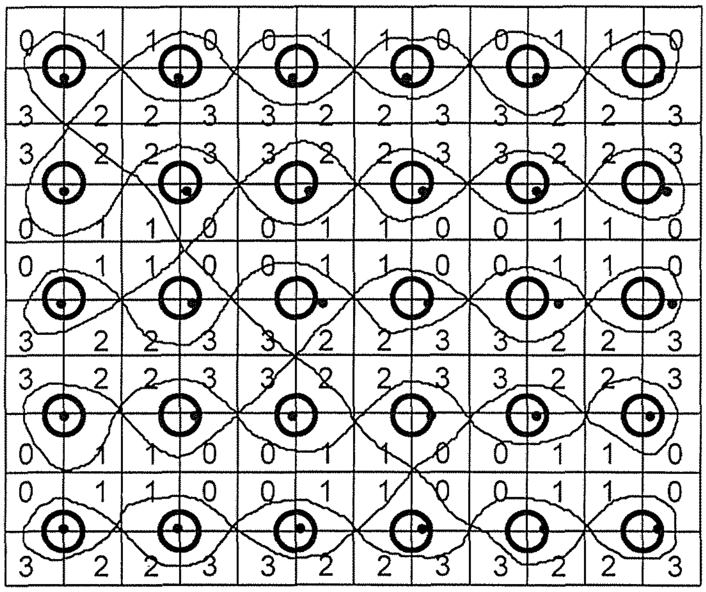
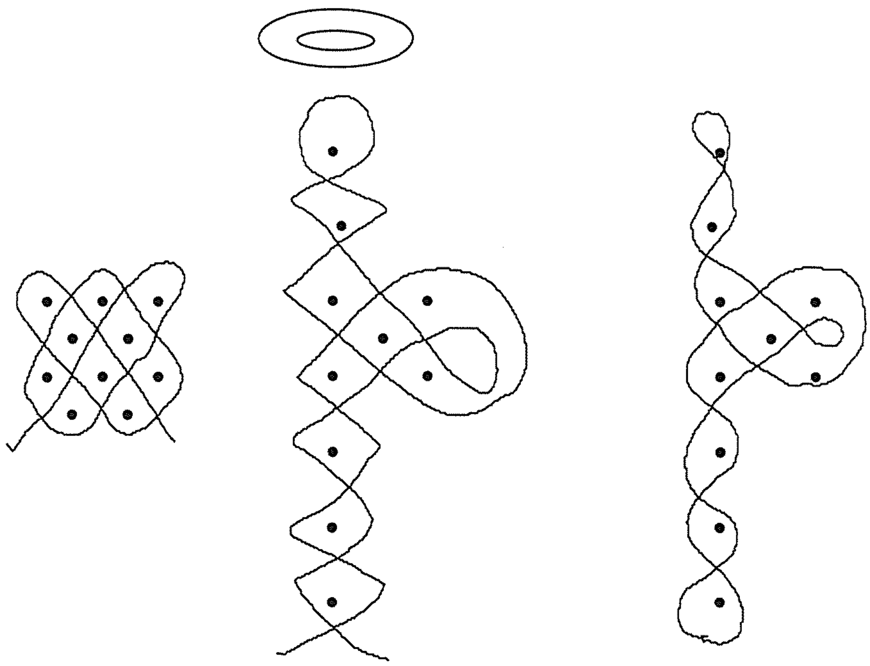


Figure 15: Attempt to integrate rows by a monolinear lusona as in linear writing, but with zero angular energy

When one tries to find a monolinear lusona which can be worked off line after line as in a linear writing, but at the same time obeys the zero energy requirement, one makes the striking experience that the outcome is an unbroken whole and every point is surrounded and held as if a human is given halt. Any region in this little space has to be minded and cared for with the utmost attention. The beauty of such lusona is that the movement locally gives any part of space a definite orientation in terms of a symmetry of $\mathbf{D}_4$ and brings on a harmonious, convective pattern of angular motion. Another surprising feature of spaces spanned by the sand tracings is their bioenergetic charge. This has already been demonstrated by the lusona of the 'joint couple'. But there exist many more drawings which carry a bioenergetic content. Consider the second and third of tracings in figure 16. The hungry orphan child (fig. 16 c) is in suspense, body hovering ungrounded. The pregnant woman (b), however, is *'bioenergetically grounded'*, she has legs. Moreover, she has an aureole so that her body appears as bioenergetically transparent.



a) b) c)

Figure 16: Some more Sona a) cock, b) pregnant woman, c) hungry orphan child

We started from the observation that space is a process of social structuration and discovered that such structuration of space is a bioenergetic process. We could not separate space from movement as we saw that movement structures space. Further we could not separate space from social facts either, since our body-language is a matter of the social context wherein we are moving. Social space, physical space and body language form a whole. That is, space is one of our institutions of signification. The spaces we believe to be in — which, as a matter of fact, we are participating parts of — determine the what and the how we experience and the what and the how we understand what and how we experience. My experience of your behaviour and my image of your experience of my behaviour involves a coupling, our experience of being together, of being a *'united couple'*, being in love or being one in our friendship. Consider the decorative pattern of some braid of the form *'vusamba'* meaning *'friendship'*. It is a monolinear lusona (figure 17).

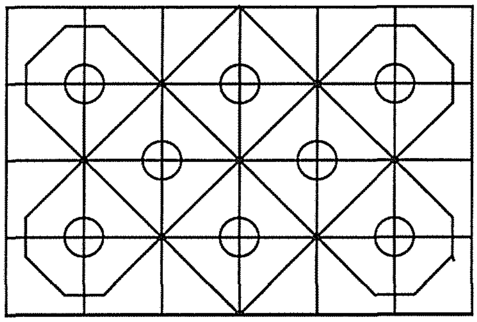


Figure 17 : Lusona *'vusamba'* meaning 'friendship'

There exist several ways in sona-geometry to extend the *'friendship'*, that is, to connect a number of such sona. Gerdes gives the example of *'mahamba já myanangana nyi ana jyenyi'* (temple of ideographs protecting notables and their descendents) from Fontinha (1983, p. 257):

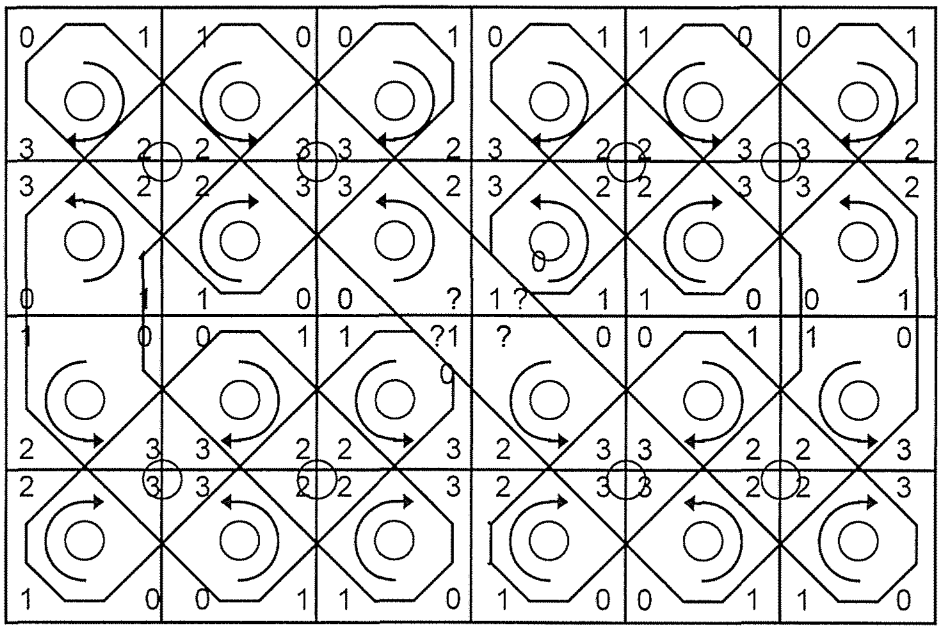


Figure 18 : Connecting 4 sona *'friendship'* (Gerdes, p. 131)

Looking at this in terms of the modulo four counting, we find out that there is some considerable 'turbulence' in the centre of the figure and around the main axis. We do not have a 'convective' flow of movement but instead quite a number of cells with equal orientation. It would be interesting to find another more symmetric solution with a harmonious zero energy pattern as we had in figure 12 ('fleeing cock'). Let us first take a look at two of the openings required to connect two sona of the type 'friendship'. This introduces an algebraic operation into the total space of sona.

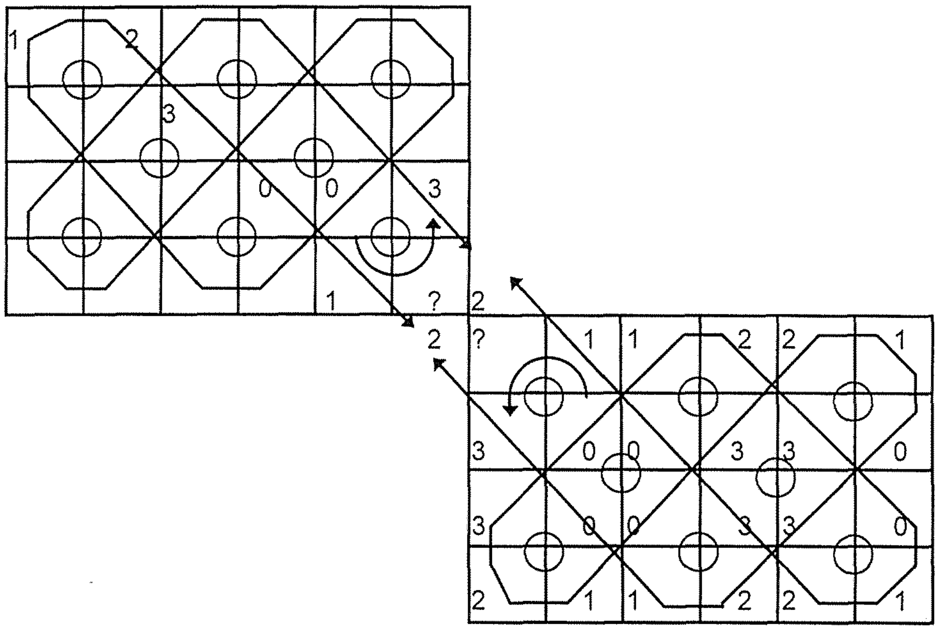


Figure 19: Equal orientations at the junction

An alternative may consist in connecting the upper left with the lower left and the upper right sona, so that we obtain

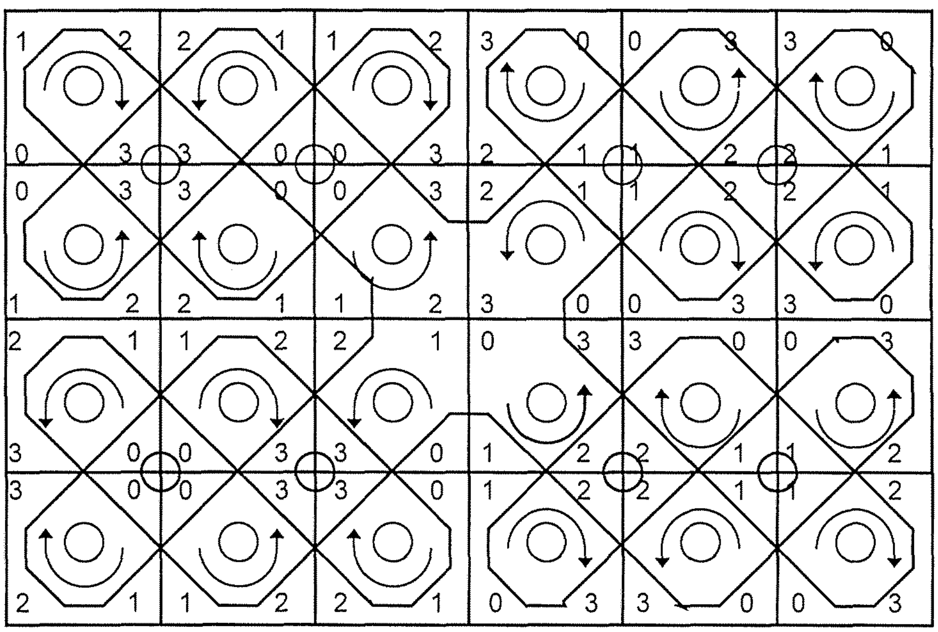


Figure 20: Turbulence in the centre caused by equal orientations

But again there is non-convective orientation at the centre and in the neighbourhood of the main axis. So this is not a solution. Maybe, we should pay attention to some left-right hand rule of combination and modify the connections at the central junction:

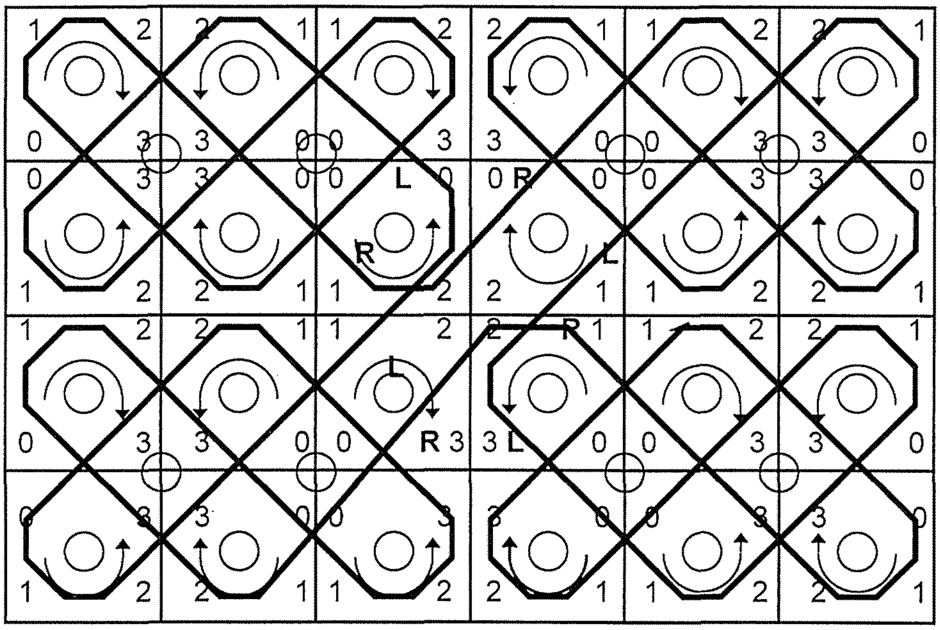


Figure 20: Harmonious 'convective' pattern of orientation

This is indeed the required solution where each clockwise locus has a counter-clockwise neighbour at the left, at the right, above and below. Each *'friendship'* is open either on the left or on the right hand-side, and any open side is linked to the open sides of two neighbours such that any left hand (L) is touching a left hand and any right hand (R) is touching a right hand. This rule of connexion reminds us of folk-dancing.

11 Concluding Remarks

Now some may say that the fundamental space is an infinite grid, some infinite algebraic group or lattice. But that would amount to the old metaphysical fragmentation of reality: here social life, there space, and the one is in the other. This is a wrong idea. However, suppose we would like to conceive a *total synchronous metaphor* for fundamental social space as is transacted by the Tshokwe sand drawings: this would have to be the *mathematical space of sona*. The reality of space as I understand it must not be confused with or reduced to its mathematical metaphor. But the second flows out of the first. Also space is both physical and social and it is constantly changing in interaction. The main thesis applying to social physical space read as follows:

[A] **Structuration of space is a bioenergetic process.**

[B] **Movement generates patterns of orientation.**

[C] **Space is a social institution of signification.**

Mathematically, orientation in space can always be established and/or changed by a minimal set of two operations. When acting in the plane those bring forth the dihedral symmetry operations of the group $\mathbf{D}_4$. Acting in Euclidean 3-space those generate the octahedral symmetries of the group $\mathbf{O}_h$, which involve period-3 rotations. Exploring social space in ethnic life, we find out that sometimes kinship relations are organized in a one-to-one correspondence with spatial orientation, that is, dihedral and octahedral symmetries (Schmeikal 1989, Ascher 1991). Several American Indian communities and segmentary societies of Australia posit very beautiful examples. Investigating different concepts of space, it would be just as interesting indeed to study American Indian Designs as Lévi-Strauss and Wilson (1984) have done. We could take into consideration the Mayan geometry or the sandpaintings of the Navajo. Braid- and Beadwork of Africa (Carey 1986) and most obviously the Tamil treshold designs (Gerdes 1989, 1997, Dutt 1925) are centred on orientation and premetric concepts of space. All those concepts have a special feature in common: they represent cultural instruments to stabilize social structures. That is, space concepts are organizers of social living. They are cognitive extensions of the meaning of body language and bioenergetic transaction. Their mathematical images may be understood as metaphors which carry some of the meaning that has to be reconstructed ever anew and in any society with peculiar means and actions.

12 Representations of Sona in Clifford Algebra

It is possible to represent a wide class of Tshokwe sand drawings in purely algebraic form. Namely, the *regular, monolinear mirror patterns* which have been defined and investigated by Paulus Gerdes (1997, ch. 6) can be set up in the Pauli algebra $Cl_{3,0}$ of the Euclidean space. This representation is simple and elegant as it is based entirely on coordinate (anti)automorphisms, that is, on the operators of the orientation symmetries of Clifford algebras (Schmeikal 1996).

The Pauli algebra is generated by three unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, which can be represented by the Pauli spin matrices σ_1, σ_2, σ_3. It is an 8-dimensional orthogonal space spanned by the units $\{1, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_{12}, \mathbf{e}_{23}, \mathbf{e}_{13}, \mathbf{e}_{123}\}$, which satisfy the anti-commutation relations $\mathbf{e}_i\,\mathbf{e}_k + \mathbf{e}_k\,\mathbf{e}_i = 0$ for $i{\neq}k$, further $\mathbf{e}_i^2 = 1$ and $\mathbf{e}_{ik}^2 = -1$ as well as $\mathbf{e}_{123}^2 = -1$. That is, the *bivectors* $\mathbf{e}_{ik}$ are hypercomplex units, the so called *quaternions*. Within the Clifford algebra $Cl_{3,0}$ we consider representations of orientation symmetries. In particular, we consider representations of the octahedral group ${}_{\delta}\mathbf{O}_h$ and the dihedral 'double-group' ${}_{\delta}\mathbf{D}_{2d} \simeq {}_{\delta}\mathbf{D}_4$ in the spin-group $\mathbf{SU}(2)$ as well as simple groups $\mathbf{D}_4$ as are generated by one pair of units $\mathbf{e}_i$, $\mathbf{e}_k$. Those groups and details about their generating basis are described in the appendix of this work and in my 1996 publication on *generation of space-time and quantum numbers of orientation.* The dihedral group $\mathbf{D}_{2d} \simeq \mathbf{D}_4$ is isomorphic with the so-called *multivector group* of the Clifford algebras $Cl_{2,0} \simeq Cl_{1,1}$ and is generated by the units

[i'] $\quad \mathbf{e}_1 \simeq C_2'$ $\qquad$ [ii'] $\quad \mathbf{e}_2 \simeq \sigma_d'$

where C_2' and σ_d' are Schönfließ symbols representing one period-2 rotation or flip and one mirror reflection (notation after Belger & Ehrenberg 1981). Thus we have an algebraic correspondence between operators of the rotation group $\mathbf{SO}(3)$ and unit vectors. The double-group ${}_{\delta}\mathbf{D}_{2d} \simeq {}_{\delta}\mathbf{D}_4 \subset \mathbf{SU}(2)$ is generated by the following minimal basis

[i''] $\quad \sigma' = (1/\sqrt{2})(\mathbf{e}_{13} - \mathbf{e}_{23})$ $\qquad$ [ii''] $\quad S_4 = (1/\sqrt{2})(1 + \mathbf{e}_{12})$

Note that S_4 which represents a period-4 rotatory reflection in the $\mathbf{SO}(3)$ has period 8 in the $\mathbf{SU}(2)$. The reason is that the $\mathbf{SU}(2)$ provides a double-cover for the rotation group $\mathbf{SO}(3)$, that is, $\mathbf{SO}(3) \simeq \mathbf{SU}(2)/\{\pm 1\}$ Finally, the octahedral symmetry ${}_{\delta}\mathbf{O}_h$ can be generated by the minimal basis

[iii] $\quad s_{11} = (1/\sqrt{2})(-\mathbf{e}_{12} + \mathbf{e}_{13})$ $\quad$ [iv] $\quad C_{24} = (1/\sqrt{2})(1 + \mathbf{e}_{13})$ $\qquad$ and -1

where s_{11} is a mirror reflection of the first of the three dihedral subgroups analogous to the Schönfließ symbol σ_d'. C_{24} is a period-4 rotation about the unit $\mathbf{e}_2$ and -1 carries out an involution of multivectors. Again, in spinor-space the Pauli matrix C_{24} has period 8 although it covers a period-4 rotation of the $\mathbf{SO}(3)$. Now we can define monolinear space tracings.

***Definition 1**: A monolinear space tracing* is given by a starting vector ξ_0 together with a finite *generative set* $W = \{\omega_i\} \subset Cl_{3,0}$ such that for all i we have $\omega_i \in {}_{\delta}\mathbf{O}_h$ or $\omega_i \in {}_{\delta}D_4$ or $\omega_i \in \mathbf{D}_4$, and it is a *loop* if

[1] $$\prod_{i=1}^{N} \omega_i = 1 \qquad \text{with } i = 1, 2, \ldots, N$$

To represent a lusona consider a rectangular grid R[n,m] as has been defined by Gerdes (1997, p. 285) but with corners in $(-m\mathbf{e}_1, -n\mathbf{e}_2)$, $(m\mathbf{e}_1, -n\mathbf{e}_2)$, $(m\mathbf{e}_1, n\mathbf{e}_2)$ $(-m\,\mathbf{e}_1, n\mathbf{e}_2)$ and grid junctions in $((2s-m-1)\mathbf{e}_1, (2t-m-1)\mathbf{e}_2)$ with s, t = 1, 2, . . ., m. Locate the origin of the Clifford algebra in $(0\mathbf{e}_1, 0\mathbf{e}_2)$ and fix the starting vector ξ_0. Now a monolinear, regular mirror pattern will cover the whole grid. That is, we will consider the latter as given by a number of N=mn orientation symmetries $W = \{\omega_i\}$. I shall denote the set W as a *generative set* refering to the associated mirror pattern. Next consider a set of *'steps'* $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{nm}\}$ as given by the recursion relations

[2] $\quad \xi_1 = \xi_0\, \omega_1,\ \ \xi_2 = \xi_1\, \omega_2,\ \ \ldots,\ \xi_r = \xi_{r-1}\, \omega_r,\ \ldots,\ \xi_{mn} = \xi_{mn-1}\, \omega_{mn}$

A *Clifford word of lenght r* is implicitly defined by the formula

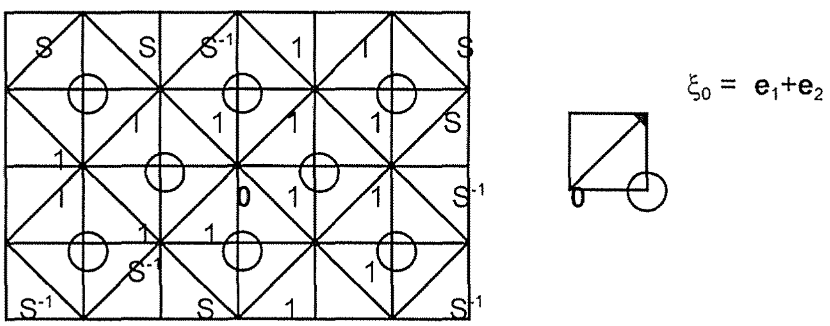
[3] $\quad W_r = \prod_{i=1}^{r} \omega_i \qquad$ with r = 1, 2, . . . , mn

So because of [2] we have to have $\xi_r = \xi_0\, W_r$. The r'th grid point on the route of the lusona is given by the sum of multivectors

[4] $\quad x_r = \sum_{i=1}^{r} \xi_r = \xi_0 \sum_{i=1}^{r} W_r \qquad$ with r = 1, 2, . . . , mn

Those are the junctions through which the lusona is passing.

Example: Lusona' vusamba' meaning 'friendship' on the grid R[2,3]



We begin to read the Clifford word W at the origin and proceed by following the route as indicated by the starting vector. Thus we have a generating set of

$W = \{1, 1, S, S, 1, 1, S, 1, 1, S, S, 1, 1, 1, S^{-1}, S^{-1}, 1, 1, S^{-1}, 1, 1, S^{-1}, S^{-1}, 1\}$

where $S = -\mathbf{e}_{12} = \mathbf{e}_{21}$ and $S^{-1} = \mathbf{e}_{12}$. Note that $S, S^{-1} \in \mathbf{D}_4 \cap_\delta \mathbf{D}_4 \cap_\delta \mathbf{O}_h$. The sets of *'steps'* $\Xi=\{\xi_r\}$ and the route $X=\{x_r\}$ of the lusona are given by the expressions

$\xi_0 = \mathbf{e}_1+\mathbf{e}_2$

$\xi_1 = \xi_0\, \omega_1 = (\mathbf{e}_1+\mathbf{e}_2)1 = \mathbf{e}_1+\mathbf{e}_2$ $\qquad$ $x_1 = \mathbf{e}_1+\mathbf{e}_2$

$\xi_2 = \xi_1\, \omega_2 = (\mathbf{e}_1+\mathbf{e}_2)1 = \mathbf{e}_1+\mathbf{e}_2$ $\qquad$ $x_2 = \xi_1 + \xi_2 = 2\mathbf{e}_1+ 2\mathbf{e}_2$

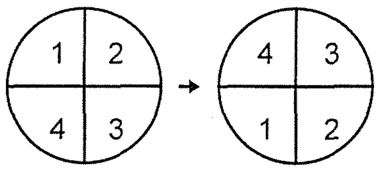
$\xi_3 = \xi_2\, \omega_3 = (\mathbf{e}_1+\mathbf{e}_2)S = (\mathbf{e}_1+\mathbf{e}_2)\, \mathbf{e}_{21} = +\mathbf{e}_1-\mathbf{e}_2$ $\qquad$ $x_3 = 3\mathbf{e}_1+\mathbf{e}_2$

and so on until to $\xi_{24} = \xi_0 = \mathbf{e}_1 + \mathbf{e}_2$, $x_{23} = 0$ and $x_{24} = \mathbf{e}_1 + \mathbf{e}_2$, which closes the loop.

Appendix: The Clifford Algebra of Orientation

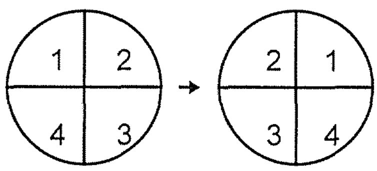
The symmetries of the dihedral group can best be represented by a quartered disk that can be turned in space about the unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, and the diagonal unit vectors $(1/\sqrt{2})(\mathbf{e}_1 \pm \mathbf{e}_2)$. The identity included, there are eight such rotations that are usually denoted by the Schönfließ symbols. Consider one such rotation about the main axis x indicated by $\mathbf{e}_1$:

$$G_1 = C_2'' = \Gamma_2$$



Here the symbol G_1 means 'first element', C_2'' is the Schönfließ symbol and Γ_2 its representation as a permutation of the symmetric group S_4. We may equally well rotate the ideogram by 180° about the vertical coordinate. In that case the quadrants 1, 4 are exchanged by 2, 3 and their order is now counter-clockwise.

$$G_2 = C_2' = \Gamma_1$$



The total set of symmetries consists essentially of proper rotations, reflections and rotatory reflections. For instance the Schönfließ symbol C_2'' denotes a proper rotation by the angle π about x, σ_d' may be regarded as a rotatory reflection of a spatial figure or mere reflection of a plane figure at the diagonal y = –x and so forth. There exist 5 symmetry axes which are shown in the following figure:

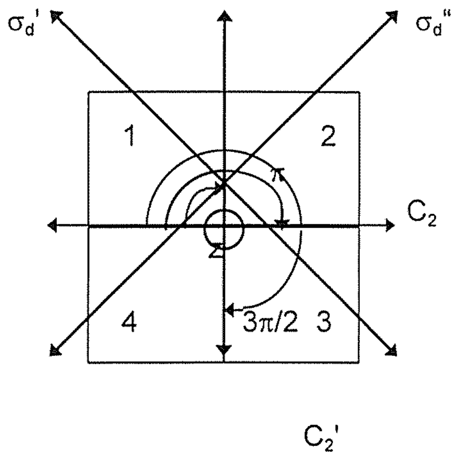


Figure 1: Rotation axes of the D_4

The Multivectorgroup of $Cl_{2,0}$ and $Cl_{1,1}$

Before we go deeper into permutations we want to show how this group can be represented within the Clifford algebra. Note, the Clifford algebra $Cl_{2,0}$ of the Euclidean plane is isomorphic with the Clifford algebra $Cl_{1,1}$. Consider the generating units $\mathbf{e}_1$, $\mathbf{e}_2$ of $Cl_{2,0}$. They generate a finite group consisting of the eight elements

$\{\pm 1,\ \pm \mathbf{e}_1\ ,\ \pm\mathbf{e}_2\ ,\ \pm\mathbf{e}_{12}\}$ subject to the following group table:

	1	e_1	e_2	e_{12}	-1	$-e_1$	$-e_2$	$-e_{12}$
1	1	e_1	e_2	e_{12}	-1	$-e_1$	$-e_2$	$-e_{12}$
e_1	e_1	1	e_{12}	e_2	$-e_1$	-1	$-e_{12}$	$-e_2$
e_2	e_2	$-e_{12}$	1	$-e_1$	$-e_2$	e_{12}	-1	e_1
e_{12}	e_{12}	$-e_2$	e_1	-1	$-e_{12}$	e_2	$-e_1$	1
-1	-1	$-e_1$	$-e_2$	$-e_{12}$	1	e_1	e_2	e_{12}
$-e_1$	$-e_1$	-1	$-e_{12}$	$-e_2$	e_1	1	e_{12}	e_2
$-e_2$	$-e_2$	e_{12}	-1	e_1	e_2	$-e_{12}$	1	$-e_1$
$-e_{12}$	$-e_{12}$	e_2	$-e_1$	1	e_{12}	$-e_2$	e_1	-1

Table 1: Multiplication table of multivector group of $Cl_{2,0}$

This table is isomorphic with that of the dihedral group $\mathbf{D}_4$. Be aware, the basis vectors of any geometric Clifford algebra do not commute. But they anticommute:

[1] $\mathbf{e}_1\mathbf{e}_2 + \mathbf{e}_2\mathbf{e}_1 = 0$

The product $\mathbf{e}_1\mathbf{e}_2$ briefly denoted by $\mathbf{e}_{12}$ is called a "bivector" or an "oriented plane area". A unit vector $\mathbf{e}_j$ squared gives 1. But a bivector behaves like the imaginary unit; its square gives −1. That is, we have

[2] $\mathbf{e}_{12}{}^2 = -1$ and therefore $\mathbf{e}_{12}{}^4 = 1$

So the operator $\mathbf{e}_{12}$ has period 4, which is typical for the dihedral group. We can rotate the ideogram about z by arbitrary multiples of $\pi/2$, which means that we apply a rotation $S_4{}^n$. The case n=1 signifies a rotation by 90° n=2 by 180°, n=3 by 270°, n=4 by 360° und n=5 by 450° and we observe $S_4{}^5 = S$. The third notation makes use of the greek symbols Γ, π and σ, which signifies the permutation representation in S_4. Since each operation results in a permutation of the four quadrants. Those are

[3]
$\Gamma_2 = C_2'' = (1\,4)(2\,3)$ $\qquad\Gamma_1 = C_2' = (1\,2)(3\,4)$
$\pi_3 = S_4 = (1\,4\,3\,2)$ $\qquad\pi_1 = S_4 = (1\,2\,3\,4)$
$\pi_2 = S_4{}^2 = (1\,3)(2\,4)$ $\qquad\sigma_1 = \sigma_d' = (2\,4)$
$\sigma_2 = \sigma_d'' = (1\,3)$ $\qquad E = (1)(2)(3)(4)$

Each symmetry operation possesses an inverse, for example π_1 has the inverse symmetry $\pi_1{}^{-1} = \pi_3 = (1\ 4\ 3\ 2)$. Using those we can give a complete list of symmetries, permutations, cycles and inverse operations of the dihedral group.

symmetry	permutation	cycles	inverse symmetry	
E		(1)(2)(3)(4)	E	
S_4	π_1	(1234)	$S_4{}^3$	π_3
$S_4{}^2$	π_2	(13)(24)	$S_4{}^2$	π_2
$S_4{}^3$	π_3	(1432)	S^4	π_1
C_2'	Γ_1	(12)(34)	C_2'	Γ_1
C_2''	Γ_2	(14)(23)	C_2''	Γ_2
σ_d'	σ_1	(1)(24)(3)	σ_d'	σ_1
σ_d''	σ_2	(13)(2)(4)	σ_d''	σ_2

The symmetries obey the following multiplication table, which is isomorphic to the Clifford algebra multiplication table of the multivector groups of $Cl_{2,0}$ and $Cl_{1,1}$.

e	π_1	π_2	π_3	Γ_1	Γ_2	σ_1	σ_2
π_1	π_2	π_3	e	σ_1	σ_2	Γ_2	Γ_1
π_2	π_3	e	π_1	Γ_2	Γ_1	σ_2	σ_1
π_3	e	π_1	π_2	σ_2	σ_1	Γ_1	Γ_2
Γ_1	σ_2	Γ_2	σ_1	e	π_2	π_3	π_1
Γ_2	σ_1	Γ_1	σ_2	π_2	e	π_1	π_3
σ_1	Γ_1	σ_2	Γ_2	π_1	π_3	e	π_2
σ_2	Γ_2	σ_1	Γ_1	π_3	π_1	π_2	e

Table 2: Multiplication table of D_4

The isomorphism between tables 1 and 2 is given by the correspondences

[4] $\{1, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_{12}, -1, -\mathbf{e}_1, -\mathbf{e}_2, -\mathbf{e}_{12}\} \simeq \{ e, \Gamma_1, \sigma_1, \pi_3, \pi_2, \Gamma_2, \sigma_2, \pi_1\}$

From the tables we can see that the period-4 operator $S_4=\pi_1$, which rotates the plane $\{\mathbf{e}_1, \mathbf{e}_2\}$ counter-clockwise by 90° about z, is represented by the bivector $\mathbf{e}_{12}$. The anticommutation relation $\mathbf{e}_1\mathbf{e}_2 + \mathbf{e}_2\mathbf{e}_1 = 0$, which is so characteristic for Clifford algebras, can be regarded as a transposition of the group theoretic inequality $\Gamma_1\sigma_1 \neq \sigma_1\Gamma_1$.

There are several possibilities to proceed, for example, with the analysis of silpa sastra:

(i) consider the spatial units $\mathbf{e}_1$, $\mathbf{e}_2$ as generators of the orientation group $\mathbf{D}_4$ in the geometric Clifford algebra $Cl_{2,0}$

(ii) represent $\mathbf{D}_4$ by the permutations Γ_1 = (12)(34) $\simeq C_2'$ and σ_1 = (24) $\simeq \sigma_d'$. Their product equals $\mathbf{e}_1\,\mathbf{e}_2 = \mathbf{e}_{12} \simeq \Gamma_1\,\sigma_1$ = (1 2)(3 4) x (2 4) = (1 4 3 2) = $\pi_3 = S_4^3$. Note, in the Clifford algebra the space area $\mathbf{e}_{12}$ is oriented counter-clockwise in accordance with the orientation of the units $\mathbf{e}_1$, $\mathbf{e}_2$ in space $\mathbf{R}^2$. Thus, while the space areas in the dandaka settlement of the silpa sastra are run through counter-clockwise, time evolution runs clockwise. Since the cycle (1 4 3 2) is exactly the order of events as described above, that is, the temporal sequence of the path of mangala-vithi surrounding the holy settlement. What is a time reversal in that context? How can we represent it? It occurs through the reversion of the bivector $\mathbf{e}_{12}$.

The orientation symmetries of Euclidean spaces R^n

(A 1) Consider **the line**

——— or symbol $\ominus$

Reverting its direction doesn't alter the figure, precisely: The orientation symmetry of R is obtained by replacing each real number r by its negative –r. The corresponding finite group is $C_2 \cong \{\pm 1\}$.

(A 2) **The plane** orientation symmetry $\mathbf{D}_{2d} \simeq \mathbf{D}_4$

Consider the plane line cross

$\oplus$

without any labels and explanatory symbols. Its space congruence group is the dihedral group $\mathbf{D}_{2d}$ acting on space or isomorphically $\mathbf{D}_4$. This is a non-commutative group with eight elements $\mathbf{D}_{2d} \equiv \{E, C_{32}, C_{12}, C_{22}, \sigma', \sigma'', S_4, S_4^{-1}\}$, where the C_{i2} are π- rotations (flips) about the vectors $\mathbf{e}_i$, the σ represent reflections and S_4, S_4^{-1} rotatory reflections (see Schmeikal-Schuh 1993). Its real irreducible **SO**(3) representations can be found in Belger & Ehrenberg (1981). The operators can essentially be represented by five π-flips about the flip-axis C_{32}, C_{12}, C_{22}, and diagonals σ', σ'', and 2 rotations by 180° about the vertical coordinate. Obviously, this group must have representations in Mat(2,**R**) (among others there is a faithful, irreducible representation with 0 and ± 1 - entries only). But what we are looking for is a double-group representation in the spin group **SU**(2), that is, in the Pauli algebra. In such a representation, the period of each operator is doubled, e. g. the group elements C_{32}, C_{12}, C_{22}, σ', σ'', S_4, S_4^{-1} will then have periods 4, 4, 4, 4, 4, 8, 8 and E will have a double with period 2, namely $\underline{E} = -1$. The double group ${}_8\mathbf{D}_{2d}$ will have 16 instead of 8 elements, but it will carry out the same transformations of space. The whole group can be generated by the minimal set

[5] 5. $\sigma' = (1/\sqrt{2})(\mathbf{e}_{13} - \mathbf{e}_{23})$ $\qquad S_4 = (1/\sqrt{2})(1 + \mathbf{e}_{12})$

From this minimal set of generators all other group elements can be calculated according to the multiplication table.[14] We have

[6] $C_{32} = \sigma''\sigma' = \mathbf{e}_{12}$ ${}_\delta C_{32} = -\mathbf{e}_{12} = \mathbf{e}_{21}$ $C_{12} = S_4\sigma' = -\mathbf{e}_{23}$ ${}_\delta C_{12} = \mathbf{e}_{23}$

$C_{22} = \sigma' S_4 = \mathbf{e}_{13}$ ${}_\delta C_{22} = -\mathbf{e}_{13}$ ${}_\delta\sigma' = (1/\sqrt{2})(-\mathbf{e}_{13} + \mathbf{e}_{23})$

$\sigma'' = C_{22}S_4 = (1/\sqrt{2})(\mathbf{e}_{13} + \mathbf{e}_{23})$ ${}_\delta\sigma'' = -(1/\sqrt{2})(\mathbf{e}_{13} + \mathbf{e}_{23})$

$S_4^{-1} = S_4^{7} = (1/\sqrt{2})(1 - \mathbf{e}_{12})$ ${}_\delta S_4^{-1} = (1/\sqrt{2})(-1 + \mathbf{e}_{12})$

${}_\delta S_4 = (1/\sqrt{2})(-1 - \mathbf{e}_{12})$ $E = 1$ ${}_\delta E = -1$

From the design of those terms, we can immediately figure out that the group belongs to the even part $Cl_3^{(0)}$. Its operators produce indeed the desired flips and rotations. Take for example $C_{32} = \sigma''\sigma' = \mathbf{e}_{12}$ with its inverse $C_{32}^{-1} = -\mathbf{e}_{12}$. It is a π-flip about $\mathbf{e}_3$ and should turn $\mathbf{e}_1$ into $-\mathbf{e}_1$, $\mathbf{e}_2$ into $-\mathbf{e}_2$ and should leave $\mathbf{e}_3$ invariant. The first means $C_{32}^{-1}\,\mathbf{e}_1\,C_{32} = -\mathbf{e}_1$ or $-\mathbf{e}_{12}\,\mathbf{e}_1\,\mathbf{e}_{12} = -\mathbf{e}_1$ which is true by conditions [1], [2] . Analogously, we can verify that σ'' commutes the basis vectors. It turns $\mathbf{e}_1$ into $\mathbf{e}_2$ and $\mathbf{e}_2$ into $\mathbf{e}_1$. Considering all the symmetries, the orientation group ${}_\delta\mathbf{D}_{2d}$ acts on the basis of $\mathbf{R}^2$ as follows:

Operators	basis vectors		signature		
E	$\mathbf{e}_1$	$\mathbf{e}_2$	+	+	
C_{32}	$\mathbf{e}_1$	$-\mathbf{e}_2$	+	–	partial
C_{12}	$-\mathbf{e}_1$	$\mathbf{e}_2$	–	+	involutions
C_{22}	$-\mathbf{e}_1$	$-\mathbf{e}_2$	–	–	main involution
σ''	$\mathbf{e}_2$	$\mathbf{e}_1$	+	+	
S_4	$\mathbf{e}_2$	$-\mathbf{e}_1$	+	–	commuted
S_4^{-1}	$-\mathbf{e}_2$	$\mathbf{e}_1$	–	+	basis
σ'	$-\mathbf{e}_2$	$-\mathbf{e}_1$	–	–	

From table 2 we can learn how an orientation group is constructed. First we allow for all possible signatures in the basis, and second we consider all possible commutations of unit

[14] This can be calculated by hand using rules (i-1) and (i-2) or by some Clifford algebra calculator such as for instance CLICAL by Pertti Lounesto.

vectors. We have 2 unit vectors, that is, 2!=2 permutations, and $2^2=4$ combinations of signs, which gives us a total of 2x4=8 elements. As we consider a double cover by elements of the spin-group, the double-group $_\delta\mathbf{D}_{2d}$ must have order 16.

(A 3) The orientation symmetry of **the space $\mathbf{R}^3$**

The orientation symmetry of the Euclidean space $\mathbf{R}^3$ contains all possible permutations of basis vectors $\mathbf{e}_i$ and all possible involutions of subspaces, with signatures of the basis running from + + +, + + – , until to – – –; so it has $3!\text{x}2^3=48$ elements. It is isomorphic with the octahedral group $\mathbf{O}_h = \mathbf{O} \times \mathbf{C}_i$ where $\mathbf{O}$ again is isomorphic with the symmetric group $\mathbf{S}_4$ and $\mathbf{C}_i$ the group of "total space inversion" in crystallography — in our terminology the *main involution* of the Clifford algebra. Since $\mathbf{S}_4$ has 4!=24 elements (permutation of 4 objects) and $\mathbf{C}_i \simeq \{\pm 1\}$ has order 2, again we end up with 48 elements. In the Pauli algebra the octahedral group $_\delta\mathbf{O}_h$ can be generated by the six reflections s_{11}, s_{12}, s_{21}, s_{22}, s_{31}, s_{32}, or by a minimal basis of generators s_{11}, C_{24}, –1, where

[7]

$$s_{11} = (1/\sqrt{2})(-\mathbf{e}_{12} + \mathbf{e}_{13}) \quad s_{12} = (1/\sqrt{2})(-\mathbf{e}_{12} - \mathbf{e}_{13})$$

$$s_{21} = (1/\sqrt{2})(-\mathbf{e}_{12} - \mathbf{e}_{23}) \quad s_{22} = (1/\sqrt{2})(\mathbf{e}_{12} - \mathbf{e}_{23})$$

$$s_{31} = (1/\sqrt{2})(\mathbf{e}_{13} - \mathbf{e}_{23}) \quad s_{32} = (1/\sqrt{2})(\mathbf{e}_{13} + \mathbf{e}_{23})$$

[8]. minimal generating basis

$$s_{11} = (1/\sqrt{2})(-\mathbf{e}_{12} + \mathbf{e}_{13}) \quad C_{24} = (1/\sqrt{2})(1 + \mathbf{e}_{13}) \qquad \text{and } -1$$

SO(3) representations of $\mathbf{O}_h$ can be found in Petraschen & Trifonow (1969).

(A 4) The orientation symmetries of **spaces $\mathbf{R}^n$**

The Clifford algebra Cl_n can also be defined on n-dimensional vector spaces $\mathbf{R}^{p,q}$ with an indefinite quadratic form $Q(\mathbf{x}) = x_1^2 + \ldots + x_p^2 - x_{p+1}^2 - \ldots - x_{p+q}^2$ with n = p + q and is denoted as $Cl_{p,q}$. Example: The Clifford algebra $Cl_{3,1}$ of the Minkowski space-time $\mathbf{R}^{3,1}$ is isomorphic with the Majorana algebra Math(4, **R**) of real 4x4-matrices. The orientation symmetries of spaces $\mathbf{R}^{p,q}$ with indefinite quadratic forms are the *hyperoctahedral* groups $\mathbf{H}_n$ having the orders $|\mathbf{H}_n| = n!\, 2^n$. Example: The hyperoctahedral group $\mathbf{H}_4$ has an order $4!\text{x}\, 2^4 = 384$ and thus contains the octahedral group $\mathbf{O}_h$ eight times, because 384 = 8 x 48.

Reconstructing Time

- Third Reconstruction -

Social Philosophy of Time

Awareness

"The doctrine that natural science is exclusively concerned with homogenous thoughts about nature does not immediately carry with it the conclusion that natural science is not concerned with sense-awareness."

Alfred North Whitehead

What is awareness? Awareness is that which is disclosed in perception through an inner sense. In perception we are aware of something which is not thought. All facts that are disclosed in awareness are impenetrable by thought, and what is immediately posited for thought in awareness cannot be explained, just as to awareness there is no further explanation. But awareness can be active in various forms.

Awareness is active in sense perception as *sense awareness.* Also it is active in practical consciousness (Giddens Bourdieu) as a recurrent movement of presence in habitual action. That is, habitus can be penetrated by *practical awareness.* It is active in body language as *body awareness* or *bioenergetic awareness.* It is active in cognition as an *awareness of thought.* Note, the *awareness of thought* is not the same as consciousness. We are aware of the content of thought by being aware of the whole extension of its signification.

Nature

What we observe in perception through the senses is nature, and nature is closed with respect to thought. Awareness in sense perception is not thought. But in awareness nature gives to thought something which is for thought only. Whitehead called this *'the factor of sense awareness'*: *"Note that it had been stated above that sense-perception is an awareness of something which is not thought. Namely, nature is not thought. But this is a different question, namely that the fact of sense perception has a factor which is not thought. I call this factor 'sense-awareness'"*. (Whitehead 1964, p. 3) In a way, nature is independent of thought, that is, it should include relations that do not require that we think about them. As

Whitehead said, we can think about nature without thinking about thought, and he called that *'homogenous thought'*. Social systems, however, are not self-contained for thought. That is, it is impossible to think about society without thinking about thought.

The Nature of Society

Here I will use the denotation of a *factor*. A *factor* is a vigilant reality. It is disclosed as a contribution to the nature of fact. Thus, a factor is a vigilant essence of fact in awareness. For instance the red of rose is a factor of the fact of rose in sense awareness. Now it seems that reality contains a *nature-culture bifurcation*. What enters experience as an essential quality by awareness, enters thought as a *bare entity*. There occurs a transposition of *sense quality* onto an *'entity of thought'*. The red of awareness is the sense quality of red and undergoes a complete loss of content when the mind transposes it onto thought. The factor of red is said to be incommunicable because sense awareness is incommunicable.

Society and nature are not related by disjunction. They are not separate. There is a factor in the *fact of social encounter* which is not thought. In social perception we are aware of something which is not thought. I call this factor *'love'*. Love can be compared with color. There are many colors: the red of rose, the red of sunset, the orange, the green blade of grass. Thus, the word *love* signifies many qualities: to have *compassion*, to *feel for* someone, a transaction of *condolence*, the *feeling responsible* not only for one's family or group, but for the whole human being. Words are not to be confused with the total meaning they are connected with. They are not the *factors* that make the *fact*. The *love* disclosed in awareness suffers a definite loss of content by its transition to the entity of cognition.[15] Namely, what is left for thought is the idea that *'this is love'*. *Love* is the nature *of society*.

Time

Time is the measure of movement with respect to the past and the future. This movement is the passage of thought. Thought operating on its passage creates the measure of time. The extension of thought beyond love gives a measure to time. The extension of thought in social structure is communication.

Communication

Communication in human society is largely based on interexperience but less on the nature of society. It is essentially a restricted form of communication which is taking place between the images we have of each other. In other words, cognition dominates sense awareness, body awareness and practical awareness. Most of that which is communicating with others most of the time, is not us but the images we have of each other. It is my image of you as

---

[15] When thought reacts to this bare entity it creates a concept, e.g. the idea of love as social structuration. Then love is explained though it cannot be explained.

your behaviour and my image of your image of me as my behaviour, my image of your reaction to my behaviour and so forth, which is communicating with a corresponding grid of images of yours but — for most of us most of the time — least of it is my body awareness, my bioenergy, my awareness in the movement of practical consciousness. That is, cognition is dominating sense, practice and body. Most of what is transacting in communication *is not our nature.*

Presence

Presence is that factor of nature which is not thought. It is disclosed in sense awareness and extends beyond nature into the nature of society. But the presence of society goes much further as it includes the other forms of awareness. Presence is not of time though its constitution is nothing other than a structuration of time, as the structure of time is disclosed in the present. But presence is not a part of time, rather the structure of time mirrors the constitution of presence. Therefore we have to inquire how the presence is constituted. First, what is presence? We are aware of presence as that which is present. What is present occurs through integration in awareness. So that which is present emerges as present in the whole domain made accessible by cognitive awareness as recognition, practical awareness and body awareness. We must not think that the present fact we are aware of constitutes an absolute object of presence. But in accordance with the natural passage of sense awareness, with the changes of body awareness, with the interactive coordination of practical consciousness, is that which is present — whether disclosed in consciousness or not — not constant, but it changes, fluctuates and is in a constant mutation. We all know of the restriction to sense awareness of the movement of presence as the 'passage of nature'. But time goes beyond that passage of nature since it involves social change. As face-to-face-contacts are modified, as coordination of positions of heads, hands, arms legs, pelvis', chests, feet moves on, as our communication of interpersonal images structurates interexperience and bioenergetic states of coupling, so the extension of presence varies and becomes structurated by the extension and coupling of awareness.

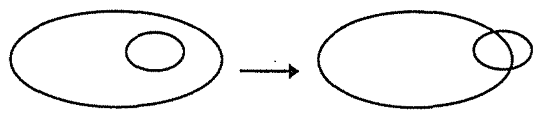
Social Theory of Time

The Lattice of time

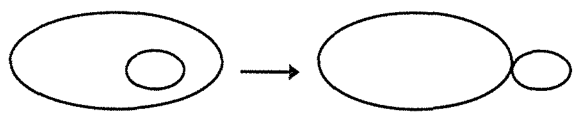
Facts of Life

Awareness is active in all the institutions of meaning: social space, body, practical consciousness and thought as discursive consciousness. The first emotional pattern of body awareness is most probably formed in a mother-child union of bioenergy. Birth sets a first strong break to the continuity of body awareness by reducing the coupling between the bioenergetic field of the mother and of the child. This sets an important mark on the scale of time which is both inner and outer as social time. Reverberating patterns of life before and after birth form partitions in the deep memory of the body which are projected onto social cognitive structures (Laing) in later life. Though this partition into pre- and postnatal structures of bioenergy may not be directly accessible to any observer's consciousness, it is well known by the body and may at any time be transposed and projected onto the conscious domain. Those transpositions and projections are most basic operations on the structure of time.

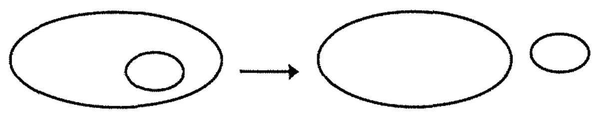
The bioenergetic pattern before birth is essentially given by the mother's body enclosing the child's body. Drawing on Whitehead: one event is enclosing the other. After birth the events are bioenergetically separate. We are therefore aware of a transition of the following form:[16]



or



or



In the first case the relation of *'inclusion'* turns into a relation of *'intersection'*; in the second *'inclusion'* turns into *'touch'*, in the third into *'separation'*. Clearly, those are mere ideal states, as complete separation is perhaps not existing. Anyway, those two states before and after birth are bioenergetically and neurologically stored as spatially different arrangements, and those are separated by spatial distance. Later we shall speculate on some details of memory formation. We shall assume that there is a tendency in nature to map relations of inclusion onto relations of inclusion and to map partitions onto partitions. In any case the temporal order of events is based on the serial accessability of arrangements of memory in space.

During some later episodes of socialization the child may internalize a manner of walking by the mother or a pattern of movement recurrent in daily practical living or some habitual form of interaction. Again those patterns are stored not only in the brain but also in the bioenergetic muscular system as components of practical consciousness. Then the young woman may become aware that she walks like her mother, that she tends to experience the same sort of stress in her daily routine and that she is just as polite as her mother and so on.

[16] Of course we do not inquire here into the complex physiological coupling between mother and child.

As she becomes cognitively aware of those facts she either may feel good and comfortable with them or she may begin to disidentify herself and try to disengage from that image. She may now create a fragmentation both outwardly in social space as inwardly in psychic space which involves interpersonal experience, cognition, feeling and bioenergy. Again this fragmentation in social space representing a fragmentation on awareness is a projection of structure on time. But the process of time has a much wider extension. Even when corporate actors stationed on different continents extend their cooperation in space and time, there arises a bioenergetic field of contacts and transactions of awareness, a structure of practical consciousness and discursive interactions which form a coherent social structure of presence and give rise to its own movement through social space. Thus time turns out to be a most vigilant lattice posed on awareness.

The Manifold of Time

The whole lattice of time cannot be comprehended on the platform of discursive consciousness alone. But it needs a medium of signification that reaches deeper than cognition. This medium is living awareness. Again the word is not the fact. The word 'awareness' can be used for example in some religious group a hundred times a day. But in that case it indicates restraint and conflict. This is not its vital social meaning. We can look at the lattice of time as a process of stratification of awareness. However, looking at it from the viewpoints of the various social strata of the science community, we can see different concepts of time. In a recent lecture Amann has investigated some of those concepts in their contemporary science historic contexts. He points out that the opinion that all time concepts have a social foundation is gradually making its way, and the idea of a definite linear time independent of social conditions is at a loss. There is *'no such self-sufficient and abstract time'*. "In 1943 Backmann ascertained: Most collegues start from the idea that time is uniform and is the same for all living organisms and non-living objects. But lately the number of biologists multiplied who from their observations and analysis concluded that living organisms have a time different from that of non-living matter, and even of different species, even single individuals have their own time — 'Eigenzeit' (1943, p. 33)

In 1988 Ewers was still able to state that the conviction that there would be only one sort of time could still dominate unchallenged in the field of natural science and philosophy, and Backmann's hypothesis remained unnoticed. Yet, it was already in 1981 that a turning point became visible, namely when Prigogine had pointed out that the world contained an infinite variety of internal times. (Prigogine, Stengers 1981, p. 264) With regard to further sociological considerations we also would like to seize a phenomenological facet of time. Almost any problem, so said Schütz, is most intimately connected with the phenomenon of time experienced (by the inner sense) which is disclosed only in stern self-consideration. It can only be on this basis that the most complex structure of concepts which are fundamental to the human sciences such as self-understanding, understanding other peoples acts, Sinnsetzung und Sinndeutung, symbol and symptom, motiv and project, Sinnadäquanz and Kausaladäquanz can be clarified. In this way, also the typical concept formations of social sciences and their exceptional attitudes towards their object can be understood (Schütz 1974, 9). Today, almost 60 years after Schütz wrote that, there has appeared the additional insight that the focus is not on mere subjective experience of time, but rather also on the social constitution of its essential components. As all the other sociological (and scientific) conceptions, so indeed the concept of time too refers to classification, division and separation, which are all categorial forms of perception and judgement fundamental for an understanding of the social world. In any case, time as a socially brought forth category of perception and evaluation posits a form of representation of the world, even in cases where it claims general liability of rational judgement.

Like power or love so is time a social manifestation; it is not a neutral quantity which poses its demands on any event. . . . "The liability of temporal orders corresponds with the convictions of the era under consideration. Thus it is clear that different competing temporal orders can coexist." (Amann 1998)

It is striking that it requires an exceptional state of awareness — stern self-consideration as Schütz said — to realise the unity in all those competing and seemingly incompatible concepts of time. Namely, they are the outcome of that process of fragmentation and structuration of awareness which is time. If we who wish to work out a meaningful concept of time are unable to trace the extension of awareness through all its institutions, we cannot possibly comprehend the whole reality of time, but we get entangled with one of its social strata of signification. In that case we restrict our knowledge to some total concept of physical time, biological time or any total concept of the genetic structuralism. But as soon as we understand the whole process of time, we begin to see how this process affects our own thought process involving recognition, practical consciousness and the subconscious as well as conscious layers of body awareness.

Thus, the present approach has a remarkable advantage. Namely, the process of time can be conceived in such a way that it is able to describe its own formation as well as the formation of its concept, the latter representing an extension of its signification into cognition. It is striking that the varying competing concepts of time are factual results of the social stratification of signification. It reflects a social structuration of awareness. That this is as it is need neither be blamed nor rejected. It is but a fact in the process of time. We may say that wherever the awareness is restricted to some small group of experts, time, as it were, turns into a petty little concept without any further commitment or relevance. But that is a bare fact which follows by necessity from the sociogenesis of time.

Relations of the Discernible

The temporal lattice possesses a velvet knot positing for us the undiscriminated social fact that something is going on. Depending on the extension of awareness our apprehension of that which goes on may be clear or puzzling or anything in between. We are in a situation, normally comprised of actors and therefore social.

We are aware of faces painted, unpainted or overroughed, of physiognomy, disguised, made up or direct or lost in thought or related in our thought to some institutional filiation, of relative positions in the immediate physical space and of relative situations in the further off social space, of feelings enclosing us in their social structure, superimposed by images evoked by some piece of news, a chatter - orientation! body moving foreward, closer to blue eyes, red lips or representative actor of CNN, click, outside in, inside out, mind cleared up. stop.

The general social encounter yields for our apprehension two components which I name in accord[17] with Whitehead the discerned and the discernible. The discerned is the field directly perceived and is comprised of those elements of the general social encounter that are directly posited as occurring in social space and directly accessible for discursive consciousness. Thereby, we do not state that all actors are conscious of the same factors and entities though they may use the same words and symbols to express the appearance of those factors and entities in thought. The entities discriminated as such individual peculiarities of the discerned are e. g. colors, smells, roughed faces, topographic features of physiognomy, emotions coming in and going out, attraction and detraction, touch sensations, a song, a talk, a story, stammering, a unity of two, a falling countenance, whining, pity, love.

There are other entities not directly disposable to the conscious mind yet attainable and disclosable by the other modalities of awareness. Those other entities are known as comprising the fragments of signification which together form the whole complex of habitus, subconscious and body memory, and are relata in relation to the entities of the discerned.

Consider a gathering of actors of different social and cultural origin. Their bearings, postures and poses, their different dialogue forms of politeness, the concessions and compromises they are ready to make are entities in the complex of practical consciousness and body language. They are relata in relation to the directly perceived social field. To be relata in relation to the discerned, it is not necessary for them to become related in cognition or discourse. A fragment of practical consciousness may relate to a fragment of body memory and both may relate to a feature directly discerned in awareness such that in the vivid social encounter an attitude towards a group of actors is formed which thereby becomes part of a cognitive image. Those entities that relate to the discerned are thus part of the discernible.

---

[17] not in full accord! The discernible of social theory includes the discernible in nature.

The complete social fact of the discernible comprises the discerned. We shall see that discerned and discernible are related by extension, and it is the extension of the discernible over the discerned which gives time a measure.

We said their relation is not bound to cognition. We know from a quite analogous situation in the philosophy of nature. "This peculiarity of knowledge is what I call its unexhaustive character. This character may be metaphorically described by the statement that nature as perceived always has a ragged edge. For example, there is a world beyond the room to which our sight is confinded known to us as completing the space-relations of the entities discerned within the room". (Whitehead 1964, p. 50) Relations of those relata from the 'exterior world' disclosed in sense-awareness do not require the expression of the fact that they are thought about. In analogy to that we can say that the relations of relata from the 'interior world' of the subconscious to the entities of the discerned do not require that they are confirmed by cognition. But they are bioenergetically present. We can keep up a transpose of Whiteheads metaphor for social philosophy: The general fact of social encounter as actually perceived and discerned has a ragged edge. The junction of the exterior of discursive consciousness and the interior of habitus, subconscious and body memory is never sharp. Subtle factors are brought in by the flow of emotions into the social domain of discursive consciousness.

The Surface

We shall conceive of cognition as the surface of signification as well as of the discurses of conscious minds as a surface in the social space of action. The junction of cognitive consciousness with the inner world comprising the embodiment of practise and the subconscious complex of the bioenergetic memory is a region of uncertainty and fuzzy intercourse of awareness. Subtle factors disclosed in body awareness float into the discursive consciousness and coordinate movements and influence verbal communication in social encounter. Metaphorically, we may therefore describe this condition by the statement that the conscious mind has a ragged edge towards the subconscious. But there is a second region in social space of discernible relata beyond the proximity of the discerned field. There are factors disclosed in sense awareness which float in from the surrounding physical space not immediately discriminated by any particular sense and not discerned by cognition within the duration of the discerned. For example, if we restrict our considerations to the perception of natural entities, there are entities further away from the adjacent field of the directly perceived which we see. We see [D1] something which we do not touch [D2] nor hear, taste nor smell. Whitehead started from the statement that [D1] and [D2] are relata in a general system of space relations.

Next, we are aware that the field of the discernible multiplies as we give up the restriction to natural entities. For instance, we look at a group of actors and we see movements as important components of body language. But we do not feel them. Now we can conceive of body experience as disclosed by an inner sense of touch. What we experience in body awareness is called emotion, sensation and feeling. Emotion, sensation and feeling are factors in the fact of body experience. The movements of the actors which we see but don't feel may induce in us a movement which we feel but don't see. But the socially discernible extends still further into the remote fields of social reality. Consider the example from the first reconstruction where an actor who is physically and bioenergetically absent because of his institutional power influences both the form of discurse and the bioenergetic states within the duration of a group present. Also there are cognitive images originating from distant substructures of the society which we may not directly discern as relata of the discernible field but which nevertheless affect the form of transactions in the present duration.

We have a general sense for the space relations disclosed by the different faculties of awareness. For instance, we are aware of the space relations between the entities disclosed in sight and those disclosed in hearing. Further we are aware of the space relations between the entities disclosed in sight and those disclosed in body experience. That is, we are aware of a relation in space between facts of sense perception and facts of the subconscious. What we are saying is that our subconscious is no less in space than is our toothache or the red of rose. This is in agreement with Whiteheads Ansatz.

The general system of space relations relating the pose of an actor — the entity discriminated by sight and cognition — is not dependent on the peculiar character as reported by our emotion. The space relations of the seen movements require an entity as relatum in the place of the emotion discerned by the inner sense of touch even although most components of their peculiar significance have not been discerned by our body experience. Thus, apart from our body awareness that entity with a specific relation to the seen movements would have been disclosed in sense awareness but not otherwise discerned in respect to its peculiar individual character.

So the conscious mind has another ragged edge towards a remote outer discernible. It is the disclosure of the remote outer discernible in terms of its relations to the discerned which defines a social place. Therefore, there is a world beyond the surface of our discursive consciousness to which the awareness is confined which is completing the relations in social space of the entities discerned in cognition.

The concept of space 'marks the disclosure in sense awareness of discernible entities known merely by their space relations'. A quite similar thought can be employed to temporal relations. Namely, the concept of a time intervall 'marks the disclosure in sense awareness of discernible entities known merely by their temporal relations to the discerned entities'. (Whitehead 1964, p. 51) But both space and time relations are derived as cognitive concepts from more general entities, which we denote as events.

Events and Durations

We discern the specific condition of a *social place* related or unrelated to individual actors through a period of time. Without specifying at present what we mean by a period of time, this is what is meant by an *event*. Events are related to each other in two ways: [E1] They are related in the active presence by awareness; [E2] They are related in nature and cognition in a way similar to set-theoretic and topological relations. As for [E1], this relation defines simultaneity with respect to a duration. We may differ between representations of those relations for one individual mind and for all minds operating under similar social conditions. For the present purpose it suffices to understand that the crude deliverance of a social relation in awareness[18] has little in common with a precise description in social theory. Such a precise theoretical description can at best be defined in terms of a 'method to approach' some vigilant fact which is never touched elsewhere but in cognition.[19] Yet the relations between events are in accord with [E2] because of the following. There is a set of events which share the duration of those events which are discerned in the awareness of that duration. This set is the general fact of both nature and social encounter now present for discernment. This set is never sharp and has no precise limits towards other durations, but it is a fuzzy set and is subject to a complex of relations which is best marked by the term of 'uncertainty'. Now an event can include another, that is, one may extend over another. They may intersect each other, that is, one event may penetrate another or 'extend into' another. Further two events may be disjoint in some specific sense though they may belong to the same duration. Finally, two events may be related by touch.

To give some examples, a train carrying a group of scientists through some tunnel to the final terminal at the Congress Hall is a social event extended over by the natural event of a mountain. Also the event of the train may extend over the toothache of one of the scientists. The Congress is another social event including that group of scientists as a smaller event. In the Congress Hall two groups with different institutional filiation may be present and thus contribute to an intersection of two events. The placenta before separating from the uterus is in a close bioenergetic contact. It is in touch with the uterus in order to be able to separate. People in love who avoid to interpenetrate each other, who do not transact ideas which would tend to conquer each other and thus cause conflict, actors in a subtle relation without conflict and exchange yet highly aware of each other are actors who are in touch.

Space and time are abstractions from the relational complex of events. Through the modalities of awareness the general fact of nature and social encounter not only becomes accessible to the mind, but also gains a unity which led us to the notations of a 'duration' and

[18] That this deliverance in awareness can be denoted as crude may signify a present cultural decay of awareness. But we have not yet gone so far with our considerations that we can give a contextual proof for this supposition.
[19] For the natural sciences Whitehead has investigated such an idealistic procedure in his lecture on the 'method of extensive abstraction'.

the relation of 'simultaneity'. All natural and social entities discernible in the present act of awareness are present and simultaneous in the whole of a duration. The relational complexes of space and time only occur through the passage of the relational complex of events.

Particularization of Transitoriness

What is past for nature is not past for society. But both nature and society are transitory termini of awareness. The quality of passage in natural and social encounter is derived from a particularization of transitoryness. In the exhibition of the passage of the general fact of social life each duration decays into another. Such general instability of durations can in no way be compared with a serial passage of time.[20] But as each act of presence is as unique as any act of awareness, so are the termini of each presence unique and alone beyond the act of cognition. That is, it is consciousness only which relates the unique termini of separate presences. The separate is thus transposed onto the distinct.[21]

Whitehead has shown that there is a procedure to construct a measurable serial time but that this does not apply to the (social) process of thought. "A temporal series, as we have defined it, represents merely certain properties of a family of durations — properties indeed which durations only possess because of their partaking of the character of passage, but on the other hand properties which only durations do possess. Accordingly time in the sense of a measurable temporal series is a character of nature only, and does not extend to the processes of thought and of sense-awareness except by a correlation of these processes with the temporal series implicated in their procedures." (Whitehead 1964, p. 66)

In social theory rather we may conceive of a lattice-like or at least a multi-serial structure of time. But to give a more relevant image of passage in social life we must not reconstruct time itself but rather the more fundamental fact of the passage of durations. First we must recollect that the terminus of each act of awareness is a unique fact of social encounter. As durations pass into one another, thought meets its liability and relates the unique termini of separate durations The events of those termini are related in the vivid presence of awareness However awareness is not stable, but is itself bound to the passage of the general social fact This implies an important statement about the stability of social durations. Namely, we have said before that events are related in nature and cognition – the first being

[20] Whitehead has cleared up this fact for nature: "The process of nature can also be termed the passage of nature. I definitely refrain at this stage from using the word 'time', since the measurable time of science and of civilised life generally merely exhibits some aspects of the more fundamental fact of the passage of nature. But what extends beyond nature to mind is not the serial and measurable time, which exhibits merely the character of passage in nature, but the quality of passage itself which is in no way measurable except so far as it occurs in nature. (p. 54f.) This serial time is evidently not the very passage of nature itself. It exhibits some of the natural properties which flow from it." (p. 65)

[21] This is the meaning of Whiteheads enigmatic statement: "Sense-awareness seizes its only chance and presents for knowledge something which is for it alone."

a terminus in sense-awareness, the second a terminus in memory-awareness – by the relation of extension. Thought fulfills its social role and extends over two distinct termini of two separate durations. Those termini, being nothing other than the discernible disclosed by two acts of awareness, comprise the discerned social encounters. Now thought has a peculiar degree of freedom (which is often decided on the ragged edge of cognition): [i] it can relate discerned to discerned, [ii] discerned to discernible and [iii] discernible to discernible. In this way it is acting on memory. (Note that case [ii] comprises two such combinations.) But thought is not impartially aware of the social entities to which it is thus related. It has a preference to relate discernible to discerned and thereby causes cognitive dissonance. To give an example: The discerned contains the image of violent actors (as a matter of fact) and the discernible contains non-violence (as a mere idea) plus some program to make actors non-violent. The intervall between selected entities from the discerned and the discernible is an expectation which, in some way, generates time. It may give rise to hope, feeling, sensation and interdependence which open a door to conflict. To express it in terms of the present Ansatz: It decomposes awareness.

It is this observation which gives us the key to understand modification of passage. The unique acts of awareness give to thought discerned and discernible entities which cognition extends over thereby destabilizing awareness. We shall call this event in the general fact of social encounter a *"decay of presence"* or alternatively a *"complexification of temporal order"*.

In 1919 Whitehead speculated: "There is no essential reason why memory should not be raised to the vividness of the present fact; and then from the side of mind, what is the difference between the present and the past?" (Whitehead 1964, p. 67)

There would indeed be no such difference. But the most essential reason why memory cannot easily be raised to the vividness of the present fact is the decay of presence. The complexification of temporal order which is brought on by the action of discursive consciousness imposes a limit to the vivid re-present-ation of memory in the act of awareness. This finally gives a possible answer to Whitehead (1964, p. 3): "It is a difficult question whether sense perception involves thought; and if it does involve thought, what is the kind of thought which it necessarily involves". In a situation where memory is raised to the vividness of the present fact cognitive dissonances and therefore thought operating on those dissonances should be minimized. That is, in a duration where the present and the past are one, perception does not involve thought. The only kind of thought which is perhaps necessarily required by the passage of awareness is that kind of thought which relates discerned events to discerned events in order to coordinate memory with the passage of nature. This may be conceived of as an extreme condition of mind as restricted to what we have denoted the *nature of society.* A condition of no-time within the variety of supposed time perceptions.[22]

Let us call to mind that the lattice of time is not restricted to the awareness of the individual actor, but that it comprises many actors who are connected in the discerned field of their mutual awarenesses. Thus, the extension of the general social encounter within a duration is related to the existence of a specific measure. Namely, the measure on a duration will have to be conceived such that it is depending on the state of fragmentation of awareness. With this statement in mind it is almost obvious that our problems can only begin. Yet, without using any exact concept of *measure on duration*, it is intuitively clear that a high degree of fragmentation of each actor's awareness is accompanied by a loss of connectivity of structure of the discerned social fact. In other words, decay of awareness as complexification of temporal order is the same as *fragmentation of social structure.*

Structures of Temporal Order

The Theory Whitehead was Urging

The strenght of Whiteheads Concept of Nature is in his compassionate admission of the dimensions of *sense awareness* and *significance* and in his radical refusal of what he called the *'materialistic theory'.* He denied the total concepts of mathematical physics as relevant structures of signification in the discernible nature by stating for example that *"there is no such thing as nature at an instant posited by sense awareness."* Whiteheads strong opposition against the mathematical physics constructed in the first half of this century has deep philosophical and sociological reasons. It is therefore that his philosophy can be extended so successfully to the field of social theory. He said:

> "On the materialistic theory the instantaneous present is the only field for the creativity of nature. The past is gone and the future is not yet. Thus (on this theory) the immediacy of perception is of an instantaneous present, and this unique present is the outcome of the past and the promise of the future. But we deny this immediately given instantaneous present. There is no such thing found in nature. As an ultimate fact it is a nonentity. What is immediate for sense-awareness is a duration. "What we perceive as present is the vivid fringe of memory tinged with anticipation. This vividness lights up the discriminated field within a duration. But no assurance can thereby be given that the happenings of nature cannot be assorted

[22] This state of experience beyond time ("Azeitlichkeit") has been called a "remarkable energy state" to which none of our concepts of time can be applied. (Fraser 1991) But as we can see from the present approach, in a refined social theory of time this state of Azeitlichkeit is not so remote. Such a condition can be brought about only by the intervention of what we have called the cognitive awareness.

into other durations of alternative families. We cannot even know that the series of immediate durations positied by the sense awareness of one individual mind all necessarily belong to the same family of durations. There is not the slightest reason to believe that this is so. Indeed if my theory of nature be correct, it will not be the case. The materialistic theory has all the completeness of the thought of the middle ages, which had a complete answer to everything, be it in heaven or in hell or in nature. There is a trimness about it, with its instantenous present, its vanished past, its non-existent future, and its inert matter. This trimness is very medieval and ill accords with brute fact.The theory which I am urging admits a greater ultimate mystery and a deeper ignorance. The past and the future meet and mingle in the ill-defined present. The passage of nature which is only another name for the creative force of existence has no narrow ledge of definite instantaneous present within which to operate. its operative presence which is now urging nature forward must be sought for throughout the whole, in the remotest past as well as in the narrowest breadth of any present duration. Perhaps also in the unrealised future. Perhaps also in the future which might be as well as the actual future which will be. It is impossible to meditate on time and the mystery of the creative passage of nature without an overwhelming emotion at the limitations of human intelligence."[23]

Orientation in the Temporal Order

The transitoriness of the creative force of existence and the permanent decay of awareness which is caused by the perpetual attempt of thought to conciliate the discerned social fact with the ideals of the discernible is at the root of our desire to bring security into the transient lattice of the passage of both natural and social events. Therefore thought has invented various procedures of abstractions to construct total concepts of order giving us the illusion in cognition that the uncontrollable passage and the particularization of transitoriness can be comprehended and given stability by those concepts. To mention a few of those some of which we shall investigate in the following considerations:

[1] serial time
[2] linear metric time
[3] lattice of time
[4] fibre bundles of multidimensional serial time
[5] mundane time
[6] quantum time
[7] relativistic time
[8] time of scale relative fractional space-time
[9] time of genetic structuralism and developmental psychology
[10] mythological time
[11] time derived by the method of extensive abstraction
[12] disconnected oriented temporal order

[23] Those are the last sentences from the Tarner lecture on time. (Whitehead 1964, chapter III, p. 72f.)

After having taken notice of this dozen of concepts let us begin with the last one which is the most natural and the most complex. It is perhaps mere fortune to observe that there are four essential topological relations of extension which we have denoted by

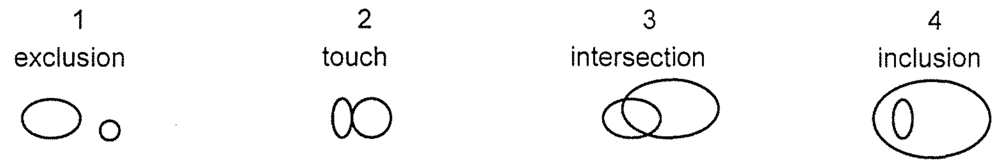


Figure 1: Definition of relations of extension

These four figures mainly exhibit the abstract fact of an event extending over other events or being extended over by other events.[24] Those four abstract termini of the relation of extension as are disclosed by awareness are now regarded as four elements of an algebra. Namely, consider transitions between pairs of the set E = {1, 2, 3, 4}, that is, elements of the abstract relation E^2. Especially we shall allow for symmetric relations between exclusion and touch, between touch and intersection and between intersection and inclusion:

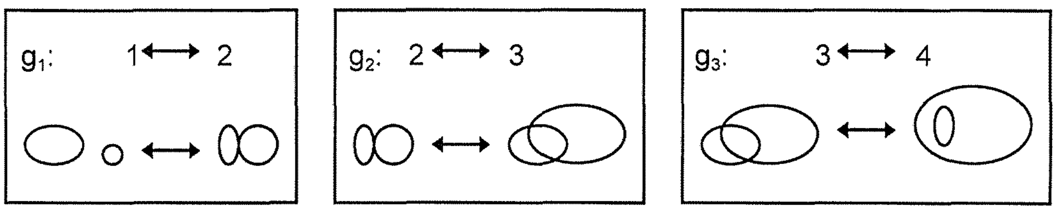


Figure 2: Generators of the algebra of extension relations

The first image (g_1) means that a separation disclosed in the act of awareness of one duration may be disclosed as a touch in another act of awareness in a second duration. The images represented show some fundamental transitions among the abstract termini of the extension relation.Those three symmetric transitions between the four termini of the relation of extension can indeed be regarded as generators of a small algebra, namely, if we represent them by the symbols of permutations. The symmetric relation g_1 will be represented by the permutation cycle (1 2), g_2 by the cycle (2 3) and g_3 by (3 4). Now all we need to do is to regard g_1, g_2 and g_3 as generators of an algebra of permutations and form all possible products between the generators and the thus obtained new elements. We obtain

---

[24] It is not by fortune that we avoid here the use of any system theoretic formalism of structuration. Merely the second relation (touch) may be comprehended in terms of the topological concept of an open or closed set. The present formalization, however, does not go beyond the limits of algebra.

the following elements which altogether form the set of permutations of the symmetric group $\mathbf{S}_4$:

[1] $\mathbf{S}_4$ = {(1)(2)(3)(4); (1 2); (1 3); (1 4); (2 3); (2 4); (3 4); (1 2 3); (1 3 2); (1 2 4); (1 4 2); (1 3 4); (1 4 3); (2 3 4); (2 4 3); (1 2)(3 4); (1 3)(2 4); (1 4)(2 3); (1 2 3 4); (1 4 3 2); (1 2 4 3); (1 3 4 2); (1 4 2 3); (1 3 2 4)}

It is striking that these permutations can be interpreted as the orientation symmetries of the Euclidean three-dimensional space. That is, the symmetric group $\mathbf{S}_4$ is isomorphic with the octahedral symmetry **O** and thus contains all possible coordinate automorphisms of Euclidean R^3 except the main involution C_i (or space inversion of crystallography), which would turn any $\mathbf{e}_i$ into $-\mathbf{e}_i$. It comprises all possible flips and rotations of periods 4 and 3 (figure 3).

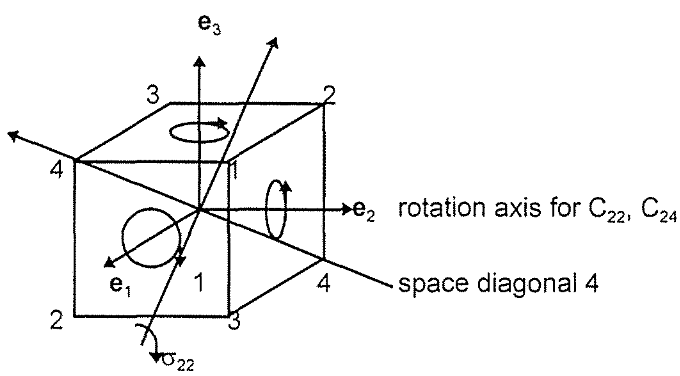


Figure 3: Orientation symmetries of Euclidean space

Namely, we can write unity = (1)(2)(3)(4); rotations by π (flips) about the main units $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are respectively

C_{12}	C_{22}	C_{32}
(1 2)(3 4)	(1 4)(2 3)	(1 3)(2 4)

Rotations by π about diagonal units of the form $(1/\sqrt{2})(\pm\mathbf{e}_i\pm\mathbf{e}_j)$ with $i\neq j$ are

σ_{11}	σ_{12}	σ_{21}	σ_{22}	σ_{31}	σ_{32}
(1 2)	(3 4)	(1 4)	(2 3)	(1 3)	(2 4)

There are 4 space diagonal units labeled 1, 2, 3, 4 of the form $(1/\sqrt{3})(\pm\mathbf{e}_1\pm\mathbf{e}_2\pm\mathbf{e}_3)$. Tetrahedral rotations by $2\pi/3$ about them are repesented by cycles of lenght 3

T_{13}	$(T_{13})^{-1}$	T_{23}	$(T_{23})^{-1}$	T_{33}	$(T_{33})^{-1}$	T_{43}	$(T_{43})^{-1}$
(2 3 4)	(2 4 3)	(1 3 4)	(1 4 3)	(1 2 4)	(1 4 2)	(1 2 3)	(1 3 2)

Period-4 rotation by $\pi/2$ about the main units $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are the cycles of lenght 4

C_{14}	$(C_{14})^{-1}$	C_{24}	$(C_{24})^{-1}$	C_{34}	$(C_{34})^{-1}$
(1 3 2 4)	(1 4 2 3)	(1 2 4 3)	(1 3 4 2)	(1 4 3 2)	(1 2 3 4)

Any of those 24 symmetry operations stands for a rotation of the **SO**(3), which brings forth a coordinate system congruent with the original one. For instance, C_{14} rotates the Dreibein by 90° about $\mathbf{e}_1$ thus turning corner 1 into 3, 3 into 2 and 2 into 4 (see figure 3), which is the cycle (1 3 2 4). The sigmas have to be decoded as follows. The first index indicates the plane to which the diagonal ratation axis is parallel. Further there are two such parallel rotation axis perpendicular to each other. For instance σ_{11} and σ_{12} are both parallel to the plane spanned by $\{\mathbf{e}_2, \mathbf{e}_3\}$, σ_{21} and σ_{22} are both parallel to the plane spanned by $\{\mathbf{e}_1, \mathbf{e}_3\}$ and σ_{31} and σ_{32} are both parallel to the plane spanned by $\{\mathbf{e}_1, \mathbf{e}_2\}$. Further, σ_{11} is perpendicular to σ_{12}, σ_{21} to σ_{22} and so on. Thus, the diagonal axis σ_{22} is parallel to the plane $\{\mathbf{e}_1, \mathbf{e}_3\}$ and runs through the midpoint of the edge 2–3. It flips corners 2 and 3, but leaves labels 1, 4 unaltered. Perpendicular to σ_{22} is σ_{21}, which flips 1 with 4, but lets 2, 3 unchanged. The σ can also be interpreted as reflections of three different dihedral groups $\mathbf{D}_4$ each of which povides the orientation symmetry of one of the planes $\{\mathbf{e}_i, \mathbf{e}_j\}$.

Several important representations of the orientation symmetries of Euclidean spaces have been discussed in the appendix of the second reconstructions. A minimal generating basis of the octahedral 'double-group' $_{\delta}\mathbf{O}$ within the spin group **SU**(2) of the Clifford algebra $Cl_{3,0}$, which we often use, is expressed in terms of the quaternions $\mathbf{e}_{ij}$:

[2] $$s_{11} = (1/\sqrt{2})(-\mathbf{e}_{12} + \mathbf{e}_{13}) \quad C_{24} = (1/\sqrt{2})(1 + \mathbf{e}_{13})$$

Consider the representations of termini 1, 2, 3, 4 (figure 1) as

$1 \ldots x_1 = \mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3$ $\qquad 2 \ldots x_2 = -\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3$

$3 \ldots x_3 = \mathbf{e}_1 + \mathbf{e}_2 - \mathbf{e}_3$ $\qquad 4 \ldots x_4 = -\mathbf{e}_1 + \mathbf{e}_2 - \mathbf{e}_3$

Using the inverse reflection $(s_{11})^{-1} = s_{12} = (1/\sqrt{2})(\mathbf{e}_{12} - \mathbf{e}_{13})$ and the inverse period-4 rotation about $\mathbf{e}_2$ given by $(C_{24})^{-1} = (1/\sqrt{2})(1 - \mathbf{e}_{13})$ we can calculate the effect of symmetry operations on the four termini. For example we obtain

$$(C_{24})^{-1}\, x_1\, C_{24} = x_2 \qquad (C_{24})^{-1}\, x_2\, C_{24} = x_4 \qquad (C_{24})^{-1}\, x_3\, C_{24} = x_1 \qquad (C_{24})^{-1}\, x_4\, C_{24} = x_3$$

which represents nothing other than the cycle (1 2 4 3) q. e. d. (see figure 1). Thus we can see that not only has each terminus of the extension relation a geometric interpretation, but also each transformation among the termini can be interpreted as the action of a symmetry operation of orientation in space. We can put this into the form of a mathematical theorem:

The relational calculus of extension has a complete geometric interpretation and can be fully represented by the orientation symmetries of Euclidean three-space.

The significance of such a proposition can be seen when we understand the following statement by Whitehead: "With this hypothesis (note: that the past partakes in the vividness of the present fact) we can also suppose that the vivid remembrance and the present fact are posited in awareness as in their temporal order." (Whitehead 1964, p. 67) The theory which I am proposing is in accord with Whitehead and is definitely denying the possibility that our remembrance can be organized elsewhere but in the present fact. So, there is no other chance for cognition than to base its image of temporal order on the space relation of extension. That is, temporal order can only be based on a calculus of space extension, whether in form of some pre-topology or pre-geometry. What I am using here are basic formulas of relational algebra and the theory of geometric Clifford algebras to demonstrate only the more fundamental aspects of such a theory of time.

The Element of Temporal Order

In order to be able to design a lattice of time we need a formal element to relate an entity of the discernible field to another such entity in such a way that this relation is unsymmetric. In what follows we shall suppose in addition that it is possible to relate whole events in such a way. That is, if one event, say E_1, can be related to an event E_2 such that $E_1 \prec E_2$ it must not be that $E_1 \succ E_2$. But it is not required that this relation be transitive. That is, from $E_1 \prec E_2$ and $E_2 \prec E_3$ it need not follow that $E_1 \prec E_3$. This is due to the fact that the events need not be connected within common durations, and the cognitive requirements for thought to establish the relation $E_1 \prec E_3$ may not be given. It is even possible that a number of events disclosed in social awareness close into a temporal cycle which then defines a mythological order of time. In a previous work (Schmeikal 1997) I have denoted this element of temporal order which allows us to project order relations onto temporal lattices the *'ordinal element'*. I first came upon the possible existence of such an *'element of temporal order'* when I began to study the striking consequences of non-commutativity of orientation, and I was confirmed in my speculation when I first read about some research findings which demonstrated that the temporal order of events can be stored within the structure of biomolecules in serial order. The non-commutativity of the dihedral group expresses the same abstract fact as the anticommutativity of basis vectors of any Clifford algebra. Roughly, the eight orientation symmetries of $\mathbf{D}_4$ are related by group multiplication such that half of the products commute and half of them do not commute. Generally for A, B $\in \mathbf{D}_4$ we may have AB $\neq$ BA. This amounts to the same as postulating that

[3] $\quad \mathbf{e}_i\mathbf{e}_k + \mathbf{e}_k\mathbf{e}_i = 0 \quad$ or equivalently $\quad \mathbf{e}_i\mathbf{e}_k \neq \mathbf{e}_k\mathbf{e}_i$

in the definition of Clifford algebras $Cl_{p,q}$. Thus we are relating the temporal order of events to the order of products having the form $\mathbf{e}_i\mathbf{e}_k = \mathbf{e}_{ik}$ of bivectors as *'oriented unit plane areas'*. This may then be generalised to multi-vectorial approaches. With regard to the relational calculus the fact that $\mathbf{e}_i\mathbf{e}_k \neq \mathbf{e}_k\mathbf{e}_i$ has no other meaning than that the temporal order once established cannot be reversed. This applies indeed only in a rather local and restricted way within some whole temporal order of discernible entities for which such relational and geometric metaphors are not valid. We shall now demonstrate some applications of irreversibility and temporal reversion respectively some of which are rather phantastic and of exaggerating generality while others are more specific but of considerable social historic factuality. In any case we can learn from those examples how the whole Ansatz is working. I will like to begin by showing a certain contrast between the old Egyptian temporal order and our present civil order of time.

Old Egyptian and Present Civil Temporal Order

Following a very fine analysis by Assman (1981) of the temporal modalities as are being reported by the science of Egyptology, I have recently worked out a parallel of the Egyptian conception to the temporal order in peoples without writing. Namely, what has been said about time in old Egypt does not differ very much from what we may call the time concepts of the *'savage mind'* (das *'wilde Denken').* To communicate a glimpse of that reality I am drawing the following relational diagram

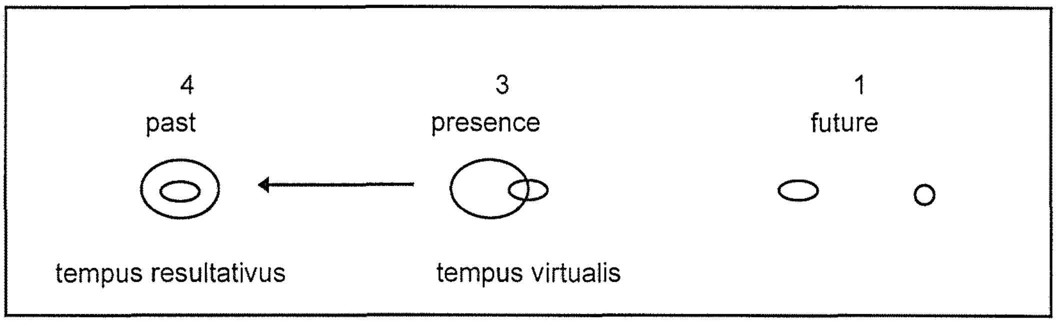


Figure 4: Old Egyptian Temporal Order

The small ellipse stands for the single actor as well as for the whole of society. The large ellipse and the small sphere signify the discernible past, present and respectively future. The Egyptian actors appear as included by the past (left side of figure 4). Their world as received from the dead god Osiris is perfect. The art of creations of their culture are perfect templates for any act of creation. Their 'present culture' as is disclosed in the vivid presence of Egyptian actors (some thousand years ago as we say) represents action aiming at a perfectum and entirely coordinated with a stream of creations directed towards the remote past. Action in presence is always incomplete (imperfectum) and comprises the virtual (tempus virtualis). It can become complete only by recreating the perfect template of the past. Therefore the actors cannot be fully included by the discerniable present. But they intersect with it. Unseen changes which arise from beyond the surface of the discursive present always point out of the past and promote a change. The living actor cannot fully approach the past. He cannot become one with the dead Osiris. It is only through mummification that his soul Ba becomes united with the time of stones — the eternal which is not outside of the world — and only now the actor becomes an Osiris himself. However, the future is unknown, and the actors are related to it by exclusion (right hand side of figure 4). That is, to act as if we could affect the future would mean to compare with the creative force. The following metaphor elucidates some important aspects of action in the old Epyptian society as well as in ethnic communities: One is not counting corns in order to control the future yield, but to distribute what is obtained (perfectum) from the past. In accord with figure 4 we can exhibit this general feature of the old Egyptian temporal order by the product of transitions (4 3) and (3 1) represented by the geometric correspondence:

[4] $\sigma_{12}\,\sigma_{31} = T_{23}$

We shall have to compare this with the following diagram which is more appropriate to describe the present civilian order of time

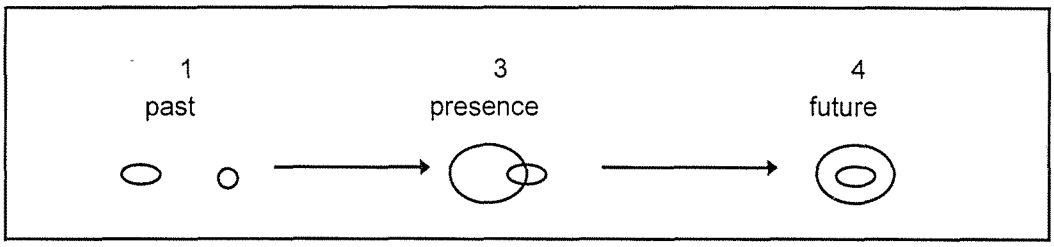


Figure 5: Present civil temporal order

As Whitehead put it, our past is gone. We seem to be unable to *raise it to the vividness of the present fact.* We are cut off it. So with respect to the past we are relata in a relation of exclusion. Our past is gone and our action appears as separate from it. Our culture as is represented in our vivid presence is entirely coordinated by a stream of creations directed towards the future. Again action in presence is always incomplete as our motives can never be fully realised and our plans never sufficiently put into realization, our wishes never entirely fulfilled and so forth. There has to be an open end towards the future, something that has to stay incomplete as it is the only source of our motivating power. So we seem enclosed by the future discernible. That is, all our actions are essentially pulled foreward by our phantasy, our images of that which will be. That is, for us the future can and must be made. This form of temporal order condenses in the abstract formula (1 3) x (3 4) = (1 4 3) or geometrically

[5] $\sigma_{31}\,\sigma_{12} = (T_{23})^{-1}$

So let us repeat we have two transitions: The one leads us from past extension relations to the present, and the other from the present to the future, and both condense in one whole which is symbolizing the temporal order.

	past relation of extension		present relation of extension	future relation of extension		
Egyptian order	σ_{12}	x	σ_{31}	$= T_{23}$		reversion of temporal order
present civil time order	σ_{31}	x	σ_{12}		$= (T_{23})^{-1}$	

The old Egyptian temporal order can be symbolized by the product of operators $\sigma_{12}\,\sigma_{31}$. These operators neither commute nor anticommute. But we have $\sigma_{12}\,\sigma_{31} + \sigma_{31}\,\sigma_{12} = 1$ and also $(\sigma_{12}\,\sigma_{31})(\sigma_{31}\,\sigma_{12}) = 1$ because $T_{23}\,(T_{23})^{-1} = 1$. Each temporal order is represented in the geometric image as a tetrahedral rotation of period 3, and each is the inverse of the other. In my 1997 monography I have described that the old Egyptian temporal order is indeed reversed relative to our present civil order of time. But we shall have to go deeper into these problems of time reversal and provide the approach with some more concrete interpretations.

Reversion of Temporal Order

Time Reversal in Mythology

There is an old Indian collection of treatises on the principles of architecture called "silpa sastra" [25] which is deeply rooted in paleolithic and neolithic mythological thought (Dutt 1925). I would like to show the reader how a reversion of time can be conceived in such a system of thought For an architect of ancient India to be instructed in the rudiments of town-planning by the silpa sastra was of the utmost importance. Composed by a mythical ancestor of architecture, a legendary sage who was familiar with the laws of cosmic construction, this knowledge goes back to Brahma himself, creator of the Universe. The groundplots of both Roman and Indian cities begin with the conception of the four main roads of the world, which indicate the four quarters of the heavens. From the East down to the West there goes the >king of roads< *rajapatha*, from South to North there leads the >broad street< *mahakalapatha*. The >path of favourable fortune< *mangala-vithi* surrounds the whole settlement. The most simple exposition of the silpa sastra concept is the *'dandaka'* (= *'commemorative of a bar'*). The dandaka settlement has 4 main doors and 4 small gates placed in the corners. Figure 6 shows what the groundplot of such a holy settlement looks like

[25] silpa = handicraft, sastra = science

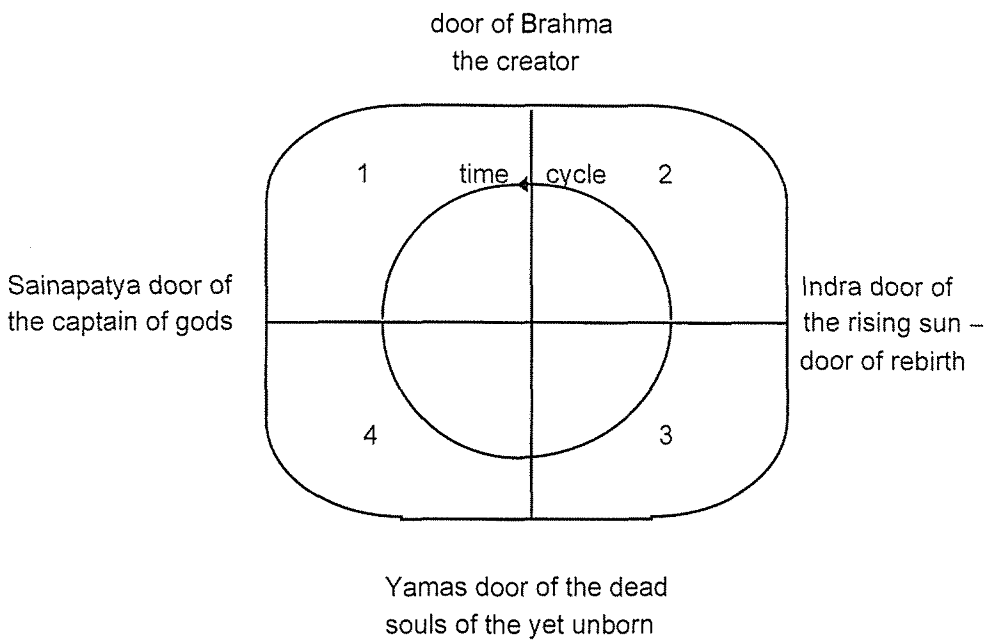


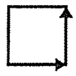
Figure 6: The holy settlement *"dandaka"*

In mythology time takes a cyclic course. To understand this consider creation in the Hinduist, Buddhist or Tantric images. Brahma gives an impuls to the wheel of life. He creates the world. But his creation is not in time. It is permanent, and time is only one of all creations. Creation, being itself beyond time, is pushing time foreward. Thinking in terms of serial progression, Brahma first creates a universe of gods necessary for the wheel of life to move on, Indra, Yamas and others and their captain god Sainapatya. Acting within the souls, their foregone emotions (in fact only 'clinging'), thoughts, words and deeds (kamma or sankara within the dependent origination paticca-samuppada in Buddhism), the psychic and mental formations are forced to reincarnate. Thus, there occurs a transition from the land of dead souls to incarnated life. We have now gone on the path of favourable fortune from Brahma to Sainapatya and from there to Yamas and further to Indra. This represents a temporal cycle of reincarnation. The revived soul lives on earth or in some other department of the universe and its ultimate aim is to become one with Brahma and (in Buddhism) to be liberated from the wheel of life. This step closes the circle.

In order to unfold the mathematical idea of this concept of time consider the spatial units $\mathbf{e}_1$, $\mathbf{e}_2$ of the orientation group $\mathbf{D}_4$ in the representation in $Cl_{2,0}$[26] also represented by the permutations $\Gamma_1 = (12)(34) \simeq C_{12}$ and $\sigma_1 = (24) \simeq \sigma_{32}$.

Their product equals $\mathbf{e}_1\,\mathbf{e}_2 = \mathbf{e}_{12} \simeq \Gamma_1\,\sigma_1 = (1\ 2)(3\ 4) \times (2\ 4) = (1\ 4\ 3\ 2) = \pi_3 = C_{34}$. Note, in the Clifford algebra the space area $\mathbf{e}_{12}$ is oriented counter-clockwise in accordance with the orientation of the units $\mathbf{e}_1$, $\mathbf{e}_2$ in space:

[26] For what follows probably the appendix of *'The Second Reconstruction'* is needed.



Thus, while the space area runs counter-clockwise, time evolution runs clockwise. Since the cycle (1 4 3 2) is exactly the order of events as described above, that is, the temporal sequence of the path of *mangala-vithi* surrounding the holy settlement. What is a time reversal in that context? How can we represent it?

Reverting the cycle $\pi_3 = C_{34} = (1\ 4\ 3\ 2)$ we obtain (1 2 3 4) which is $\pi_1 = (C_{34})^{-1}$. According to the correspondences in the $Cl_{2,0}$-representation this equals $-\mathbf{e}_{12} = \mathbf{e}_2\, \mathbf{e}_1 \simeq \sigma_1\, \Gamma_1$. So the sequence of unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$ is reverted. In this way time reversal is obtained from a reversion of a space area or bivector $\mathbf{e}_{12} \to -\mathbf{e}_{12}$. But this is essentially what we are doing in the Minkowski space-time when we carry out a transition from $\mathbf{e}_4$ to $-\mathbf{e}_4$.

We know that in the Minkowski space, time $\mathbf{e}_4$ represents a temporal unit vector. Therefore $\mathbf{e}_4$ squared gives -1 instead of $+1$. (That is, it is usually represented by a complex or hypercomplex quantity.) The same holds for the bivector $\mathbf{e}_{12}$. As soon as we use the isomorphism $Cl_{1,1} \simeq Cl_{2,0}$ the correspondence is complete. In that case the bivector $\mathbf{e}_{12}$ is replaced by the temporal vector ε_2, and the generating units obey the equations

[6] $\varepsilon_1^2 = +1$ and $\varepsilon_2^2 = -1$

From these equations there follows that $\varepsilon_{12}^2 = \varepsilon_1\varepsilon_2\varepsilon_1\varepsilon_2 = -\ \varepsilon_1\varepsilon_2\varepsilon_2\varepsilon_1 = \varepsilon_1^2 = 1$ because of anticommutation. Thus, the unit bivector ε_{12} in $Cl_{1,1}$ actually plays the role of the vector $\mathbf{e}_2$ in $Cl_{2,0}$. After all, from these correspondences there follows the isomorphism $Cl_{1,1} \simeq Cl_{2,0}$. So it has been shown that in the above system of mythology time reversal can be represented by a reversion of an oriented space area or some 'hypercomplex unity' respectively. We can state the following analogy of transitions

Physics:	**Sociology:**
$\mathbf{e}_4 \to -\mathbf{e}_4$	$\mathbf{e}_{12} \to -\mathbf{e}_{12}$
proton $\to$ neutron	Indra door of incarnation $\to$
time reversal $\to$ charge conjugation	$\to$ Yamas door of the dead
and parity flip	incarnation $\to$ discarnation

The Old Egyptian Temporal Order

We have said that any cognitive image of temporal order has to be led back to the order of space and that time reversion can be based on the reversion of a hypercomplex number.

Now consider for just one more instant the temporal order of the old Egypts. We could represent it by a tetrahedral geometric operator of the form

$\sigma_{12}\,\sigma_{31} = T_{23} \quad \in Cl_{3,0}$ see [4]

This quantity is known to be equal to

$T_{23} = ½\,(1 - \mathbf{e}_{12} + \mathbf{e}_{13} + \mathbf{e}_{23})$

and is built up by the three quaternions $\mathbf{e}_{12}$, $\mathbf{e}_{13}$ and $\mathbf{e}_{23}$. It is excactly when we revert those three bivectors and turn $\mathbf{e}_{12}$ into $\mathbf{e}_{21}$, $\mathbf{e}_{23}$ into $\mathbf{e}_{32}$ and $\mathbf{e}_{13}$ into $\mathbf{e}_{31}$ that the temporal order is reverted, namely

$(T_{23})^{-1} = ½\,(1 + \mathbf{e}_{12} - \mathbf{e}_{13} - \mathbf{e}_{23}) = T_{23}^{\sim}$ where ~ stands for Clifford algebra reversion.

Note that in section 3.2.2 the rotation operator $T_{23}^{\sim}$ represented the present day civil order of time. Being aware of those three cases shown we are now ready to anticipate a natural supposition of any geometric representation of temporal order, namely, that time reversion can be based on oriention of the underlying geometric Clifford algebra.

Temporal Order in Social Kinship Relations

Exercise of power, delegation of power, social exchange of any form, friendship, match-making, mating and marriage, touch, copulation, conception, nidation, pregnancy, birth and death are lively examples of relations of extending over and being extended over. It should be rather impressive to understand the action of social space and temporal order as bringing about and being brought about by social ties and congenial relations. But most natural and social laws operative in this domain seem to have been lost and foregotten. There are however a few ethnic groups and settlements of peoples without writing where some of the relations between physical and social space are still known and where social, spatial, temporal and ritual order of action form an indivisible whole and thus define a mythical form of life. With this in mind I have analysed some social structures as have been described by ethnographers (Schmeikal-Schuh 1989). I am not saying that the ethnic groups about whose kin-structure I am now going to reflect are representative for the whole sample of ethnology. But I believe that they have realised best what we may call a unity between social and physical time-space. If I only convey the meaning of such unity my examples will perfectly accomplish their purpose. In those examples under consideration the main entities are settle-ments, actors, groundplots, clans, women, men, ceremonies, mothers, fathers, grandparents, uncles and children, and the main events are mating and marriage, filiation, residing, patricirculation, exogamy and so forth. What we investigate in terms of these entities and events is the temporal order as is given by the passage of generations and embedded into a structure of phratries, clans and social strata. This comprises so many constitutive events that it is hardly possible to trace the whole hierarchic lattice of extension relations involved. For example the mating- and marriage-relation requires that there are relations of touch between clans. One clan, often that of the male, will for some period of time extend over the clan of the woman so that the mating of the couple becomes a legitimate social act. The couple will have to go through a series of relations of touch and playing before they become a united couple. Last not least are marriage and copulation social relations of extending over and being extended over. Clearly, the whole lattice of relations involving inclusion, touch, intersection and exclusion as disclosed in the actors' awareness cannot be described in its totality. All we can do is to use thought to select the most important and obvious constituents of kinship and find out about their social temporal and spatial order. Now it is not entirely accidental that we have made use of a specific correspondence between the termini of extension and space symmetry operations of orientation. Namely consider by the guide of mere intuition a sequence of figures like that one

4	2	3	1
inclusion	touch	intersection	exclusion

We may visualize in it an abstract image of child-birth, and therefore I will call this cycle (1 4 3 2) the *'motherhood extension'* and abbreviate it by the letter 'm'. A special bioenergetic awareness may lead us to the insight that in touch our body awareness may experience a sudden transition into a relation of inclusion. There is a sudden turnover from touch to intimacy and feeling surrounded by the other. Ronald Laing reported about the wish of women to feel surrounded and covered by their men in order to be able to surround and cover their children. I shall call this relational transition (2 4) from touch

 to inclusion 

a *'father extension'* or briefly 'f'. It is clear that on this basis we can form products such as m^2f or fm meaning *'grandmother of father'* and *'father of mother'* respectively. The specific notation does not matter, but it is perhaps worth mentioning that the mathematical analysis of the following case as has been reported by Laughren and thereafter by M. Ascher most naturally leads to that notation. Perhaps it is also useful to first provide the reader with several isomorphic representations of the dihedral symmetry group $\mathbf{D}_4$ and with an appropriate multiplication table

E	(1 4 3 2)	(1 3)(2 4)	(1 2 3 4)	(2 4)	(1 4)(2 3)	(1 3)	(1 2)(3 4)
$\underline{1}$	C_{34}	C_{32}	$(C_{34})^{-1}$	σ_{32}	C_{22}	σ_{31}	C_{12}
$\underline{1}$	$S_4^{\ 3}$	$S_4^{\ 2}$	S_4	σ_d'	C_2''	σ_d''	C_2'
$\underline{1}$	π_3	π_2	π_1	σ_1	Γ_2	σ_2	Γ_1
1	e_{12}	-1	$-e_{12}$	e_2	$-e_1$	$-e_2$	e_1
	m	.	.	f	.	.	.

Table 1: Ddifferent notations of isomorphic representations of the dihedral group D_4

Remark: First row are permutation cycles, second row are operators of the octahedral group O (see section 3.2), third and forth row is the notation used by Belger and Ehrenberg (1981) and Schmeikal (1993), symbols in the forth row denote permutations, fifth row are the basis vectors of the Clifford algebra $Cl_{2,0}$, row 6 is to be worked out.

e	π_1	π_2	π_3	Γ_1	Γ_2	σ_1	σ_2
π_1	π_2	π_3	e	σ_1	σ_2	Γ_2	Γ_1
π_2	π_3	e	π_1	Γ_2	Γ_1	σ_2	σ_1
π_3	e	π_1	π_2	σ_2	σ_1	Γ_1	Γ_2
Γ_1	σ_2	Γ_2	σ_1	e	π_2	π_3	π_1
Γ_2	σ_1	Γ_1	σ_2	π_2	e	π_1	π_3
σ_1	Γ_1	σ_2	Γ_2	π_1	π_3	e	π_2
σ_2	Γ_2	σ_1	Γ_1	π_3	π_1	π_2	e

Table 2: Multiplication table of D_4

Next consider a settlement of some Brasilian tribe (Bororo) or of Australian aborigines (Walpiri) with a roughly spheric groundplot. It may be subjected to a very complex structuration process, which for the present we need not go into, and it has eight clans arranged above and below some invisible diametrical line separating the exogamous halves (figure 9). The marriage pattern is as follows:

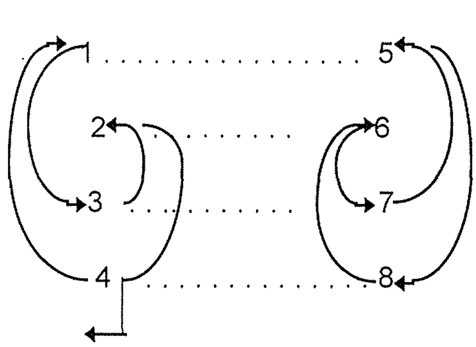


Figure 7: Marriage and kin-ties in a spheric settlement

A woman from the first clan is allowed to mate a man from clan 5. By rule their children become members of clan 3. A girl from clan 3 marries a man from clan 7 and their children belong to clan 2 and so forth. Generally, men and women from clans 1, 2, 3 and 4 marry women and men from clans 5, 6, 7 and 8 respectively. When a woman from clan 5 marries a man from clan 1, their children belong to clan 8 and so on. Now the whole structure can be decomposed into 2 matricycles and 4 patricycles (figure 8).

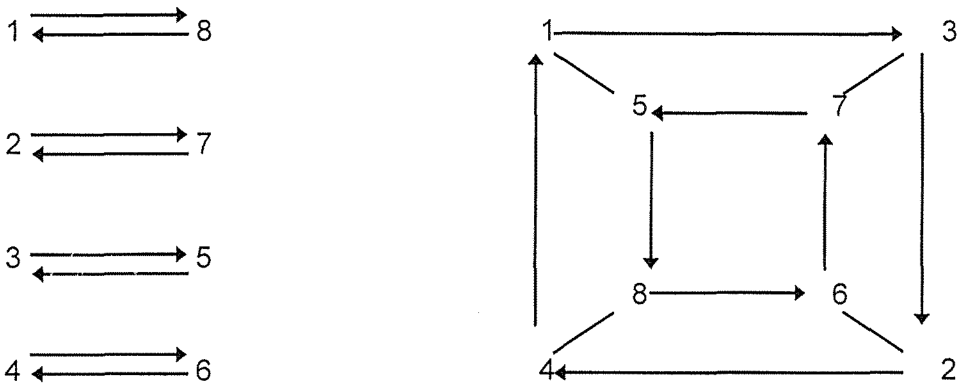


Figure 8 a: Patricycles of period 2; b: Matricycles of period 4

Figures 7 and 8 are essentially equal to figures 3.1 and 3.2 from Ascher (1991, p. 71), who refers to Mary Laughrens' (1982) '*Walpiri kinship structure'*. Such figures are often exhibited in books on mathematical anthropology or current ethnomathematics. They are very useful as they bring the whole relational structure into a simple diagram. But it is also remarkable that very often they do not immediately show us the connection with geographic matters, the regions in the groundplot where the clans are located. They even often represent a muddle. We shall see that the present diagrams (figures 7, 8) somehow muddle up the space- and time relations and leave us with a riddle. It is only the present Ansatz of geometric algebra which allows us to solve that riddle straight away. Namely, how are the clans arranged on the ground plot?

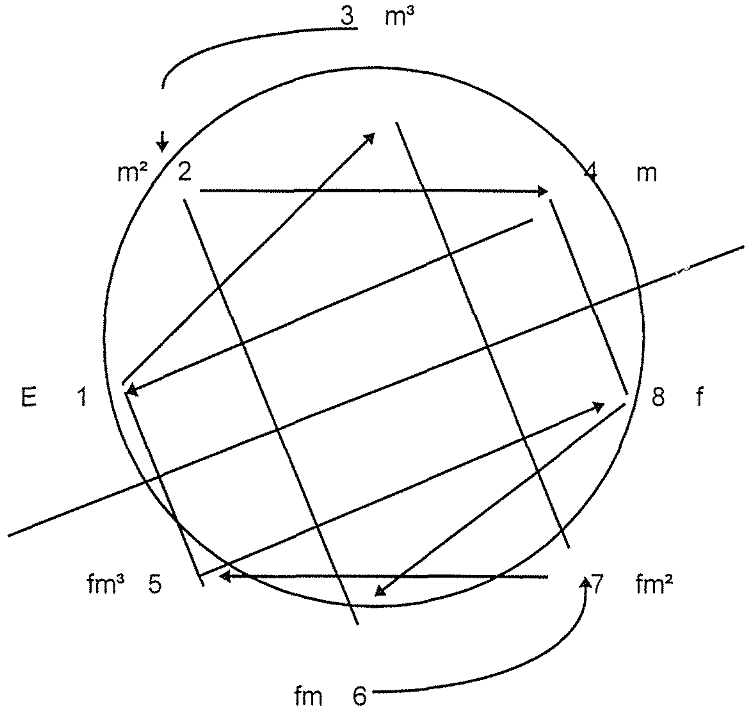


Figure 9: Kin-structure on a spheric groundplot

The interpretation is easy to work out and follows a well known procedure. We start off by ascribing to clan 1 the unit element of the algebra. Next, we are aware that mothers of people from clan 1 are residing in clan 4. Therefore, we coordinate clan 4 with 'm' being C_{34} = (1 4 3 2) = π_3. Mothers of mothers of people in clan 1 are in clan 2, and grandgrandmothers are in clan 3, in accordance with figure 7. It is only the forth generation of mothers who relative to someone in clan 1 return to 1. Therefore, we call this cycle (1 4 3 2) a matricycle, and it is in perfect agreement with the laws of algebra that the operators m, m^2 and m^3 are respectively coordinated with clans 4, 2 and 3. The algebraic equations associated with this are quite obvious, namely we have

[7] $m = C_{34} = (1\ 4\ 3\ 2) \simeq \mathbf{e}_{12}$; $m^2 = (C_{34})^2 = C_{32} = (1\ 3)(2\ 4) \simeq -1$;

$m^3 = (C_{34})^3 = (1\ 2\ 3\ 4) \simeq -\mathbf{e}_{12}$ and $m^4 = 1$

(use table 1 to understand the correspondences!)

The 'mother-operator' m is a period-4 bivector or (hyper)complex unit. That is, it behaves like the imaginary number i: $i^2=-1$, $i^3=-i$ and $i^4=1$. The same holds of course for the quaternions j and k or the *'director'* $\mathbf{e}_{123}$ of $Cl_{3,0}$ as also for the **SU**(2) representation of C_{34} as a spin-matrix in the Pauli-algebra:

[8] $C_{34} = (1/\sqrt{2})(1 + \mathbf{e}_{12})$

There are indeed infinitely many ways to represent 'm' in geometric algebras. But we are now ready to carry out the next step and coordinate the clans of the remaining half of the village with specific orientation symmetries. Therefore, we procede as follows: the husbands of mothers of people from clan 1 are in clan 8. They are the fathers of people from clan 1. Therefore we coordinate clan 8 with the symbol 'f'. Next, we know that the mothers of those fathers are in clan 5. So we coordinate clan 5 with the product 'mf' and go a step further, namely, the grandmothers of those fathers are in clan 7 and so clan 7 becomes an 'm^2f'. But beware, there is a second notation providing us with a kinship-relation of clan 1 with the relata in clan 7: Take a look at the husbands of grandgrandmothers m^3 (residing in clan 3). Definitely they are the fathers of the grandmothers in relation to 1. Therefore, clan 7 (where the husbands of the grandgrandmothers are residing) collect the fathers of grandmothers in relation to 1 and are therefore coordinated with the product 'fm^2'. So we obtain a first equation which contributes to the identification of the algebra. We have to have $m^2f = fm^2$. We said that in clan 5 are the mothers of fathers relative to 1, that is, 'mf'. But clan 5 also collects the husbands of the forth generation of mothers. Those are the fathers of the third generation of mothers 'fm^3'. So we have a second equation: $mf = fm^3$. There is now just one clan left for identification. This is clan 6. Observe the matricycle in the 'lower' half of the groundplot. In relation to the fathers from clan 8 women in clan 6 must be mothers of mothers of mothers. Thus, in relation to clan 1 they are 'm^3f'. Finally, let us be aware that clan 6 also collects husbands of grandmothers (from clan 2), so that clan 6 is also coordinated with 'fm'. The third equation we have thus obtained is $m^3f = fm$. Now we can form all possible products between any pair of those eight 'numbers' which altogether read $\{1, m, m^2, m^3, f, fm, fm^2, fm^3\}$. To give a few examples: $m^2(fm) = (m^2f)m = fm^2m = fm^3 = mf$ or $m^3(fm^3) = (m^3f)m^3 = (fm)m^3 = fm^4 = f$. The result is that any number of the set of eight multiplied with any other gives back one of the eight. Also each of the eight numbers has an inverse, e. g. m^2f must have an inverse. First note that $m^3f = fm$. Also we know from the fact of patricyclicity that $f^2 = 1$, which means that in relation to clan 1 the grandfathers are back in clan 1. Therefore we consider the product m^3f as inverse to fm since we have $(fm)(m^3f) = f^2 = 1$. Finally, what is the inverse to m? This is m^3 and so on. Those are the reasons why the set of eight forms a group, and Marcia Ascher pointed out correctly that they form a representation of the dihedral group $\mathbf{D}_4$. The groundplot now looks as shown in figure 10:

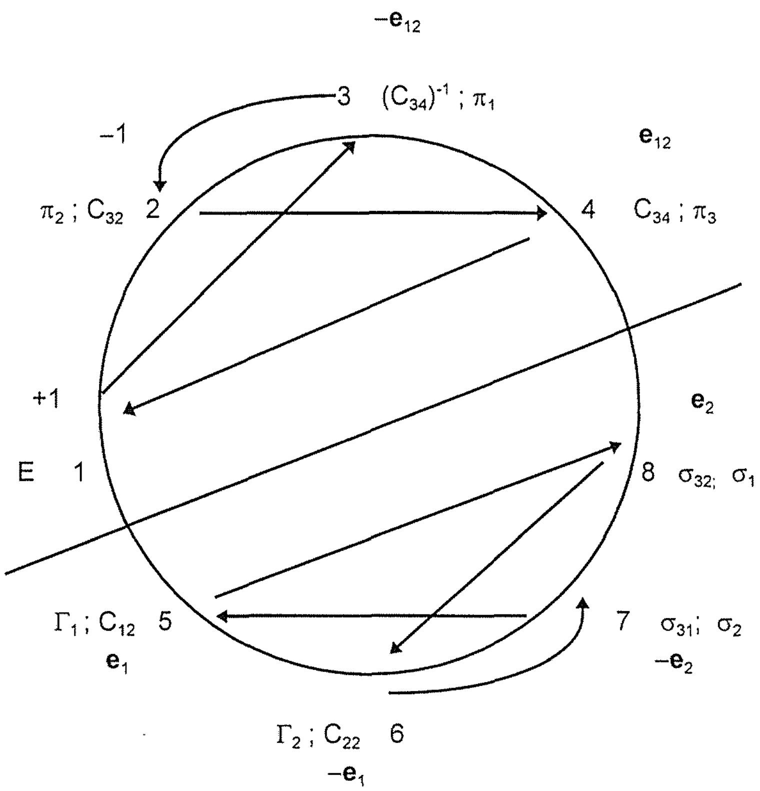


Figure 10: Orientation symmetry operators of kin-structure

Now we have purposely coordinated the operators of $\mathbf{D}_4$ with the basis of the plane Clifford algebra $Cl_{2,0}$, which as we have seen is also a representation of $\mathbf{D}_4$ (again use table 1 in order to verify the correspondences!), and the whole thing seems to be somewhat muddled up. Obviously the structure from figure 7 does not precisely correspond with the geometric idea of the plane. Well, the situation is not hopeless. Let us merely set the dislocated bones, after all that is the meaning of algebra!

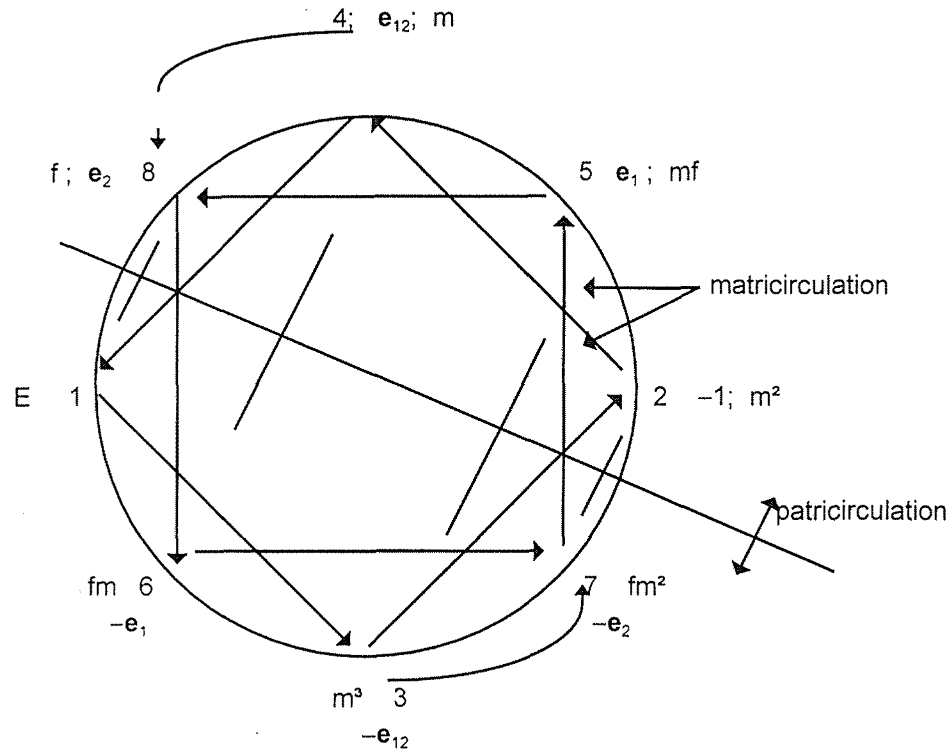


Figure 11: Algebraically corrected kin-structure

Figure 11 discloses that this structuration of kinship is perfectly coordinated with the orientation symmetries of the plane. It is striking that the 'old' diameter which separated the village into two exoganous halves does no longer exist. But there is now a diameter which separates fathers from sons If the sons would wish to visit their fathers, they would have to cross the village eventually by passing through some central place and they could do that hand in hand with their grandfathers. It seems that the attempts to interpret the partition of such kin-systems in terms of exogamy does not reflect the original structuration process. This can better be described by the statement that none of the parents, aunts uncles and first cousins reside in one of those clans into which one is supposed to marry and no son resides in the clan of his father but fathers and sons define a regional partition of the village into halves. Obligations and actions in the vivid presence extend into the remote past as also into the future. It may be possible that the father/son separation may contribute to the arisal of regional exogamy The considerate reader will probably have realised that the algebraic image we have posited unites the vector plane with the complex plane. This is a nice feature of the Clifford algebra $Cl_{2,0}$. The vector plane is the *odd part* of $Cl_{2,0}$ and has indeed only one point in common with the complex plane being the *even part* of $Cl_{2,0}$, namely zero. One matricycle belongs to the vector plane (earth), the second matricycle belongs to the complex plane (heavens). Now there seems to remain one question unanswered: What brings forth the temporal order in this case, and why is the future different from the past if it is different? Consider the operators m and f and recall we said the element of temporal order (section

3.2.1 and equation [3]) is to be based on the non-commutativity of orientation symmetries. It is near at hand to regard the following equation as such a fundamental relation

[9] $\quad fm \neq mf,$

which means that the clan of the father of mother is not the clan of the mother of father. Algebraically,

[10] $\quad \mathbf{e}_{12}\,\mathbf{e}_2 \neq \mathbf{e}_2\,\mathbf{e}_{12},$ which is equivalent to [3].

That is, saying that the bivector $\mathbf{e}_{12}$ does not commute with the vector $\mathbf{e}_2$ amounts to the same as saying that the vector $\mathbf{e}_1$ does not commute with $\mathbf{e}_2$. Namely, we have

[11] $\quad \mathbf{e}_{12} = -\,\mathbf{e}_{21},$ which is $\quad \mathbf{e}_1\,\mathbf{e}_2 + \mathbf{e}_2\,\mathbf{e}_1 = 0$

the usual anticommutativity of basis vectors in Clifford algebras. Substitution of [11] into [10] gives $\mathbf{e}_1 \neq -\mathbf{e}_1$, which is the exact algebraic transposition of equation [9]. Now we can answer both questions that we have asked further above, namely, why is there a temporal order in the Walpiri kinship structuration? The answer must be that the clan of the mother's father is not the clan of the father's mother. But in the matricycle clans 5 and 6 are two generations apart. The grandmothers of the people in clan 5 (mf) are in clan 6. What does it mean in this connection that time goes on and that as time is going on there is constituted a differentiation between the clans of the mothers of fathers and the clans of the fathers of mothers? It means nothing other than that $\mathbf{e}_1 \neq -\mathbf{e}_1$. Thus, once more:

Why is time going on?
the answer: Because the East is not the West!

Let us finally consider time-reversal in such a system of kinship structuration. We can pose the question even a third time: Why is time going on? And the answer is now: Because the succession of generations is arranged in space as a geographic arrangement of matricycles. Clearly, time is reverted as soon as we revert the cycle (1 4 3 2) and turn it into its inverse (1 2 3 4). This is the same as turning a transition from 'mother' to 'grandgrandmother' into the inverse transition from 'grandgrandmother' to 'mother' or a turnover from the complex unit $\mathbf{e}_{12}$ to $\mathbf{e}_{21}$. This is nothing other than the reversion operation ~ in the Clifford algebra $Cl_{2,0}$.

Temporal Order in a Tshokwe Sand Drawing

We shall go into a very last example of time reversion and once again point it out that movement locally constitutes orientation of space. By reverting this movement the orientation is also changed. Once the temporal order of movement is connected with the orientation of space it can often be represented by the orientation of hypercomplex units of the fundamental geometric algebra. Thus, a reversion of time is connected with a reversion of the Clifford algebra or more generally a change of the pattern of orientation. The following example is such a one which is a little more complex. Consider the sand drawing of the fleeing cock of the Sona tradition. Here not only is the sense of rotation reverted but there is also a reflection occurring. Namely, first any $(C_{34})^{-1}$ is turned into a C_{34} by multiplication with C_{32} and next a reflection σ_{32} brings forth the time-reverted route. Note that we have $\sigma_{32}\, C_{32}$ equals σ_{31}.

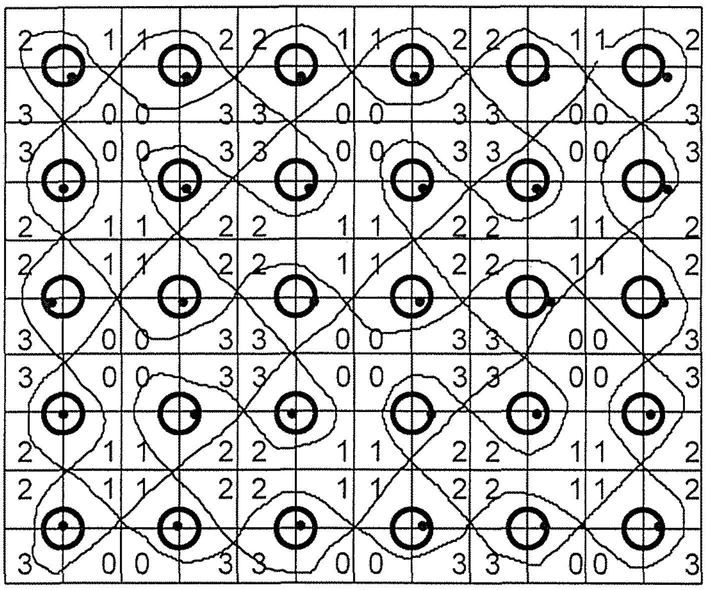


Figure 12: Orientation structuration of lusona 'fleeing cock'

This is the pattern

σ_{31}	$(C_{34})^{-1}$	σ_{31}	$(C_{34})^{-1}$	σ_{31}	$(C_{34})^{-1}$
C_{34}	σ_{32}	C_{34}	σ_{32}	C_{34}	σ_{32}
σ_{31}	$(C_{34})^{-1}$	σ_{31}	$(C_{34})^{-1}$	σ_{31}	$(C_{34})^{-1}$
C_{34}	σ_{32}	C_{34}	σ_{32}	C_{34}	σ_{32}
σ_{31}	$(C_{34})^{-1}$	σ_{31}	$(C_{34})^{-1}$	σ_{31}	$(C_{34})^{-1}$

By reverting the route of the cock (see second reconstruction) this pattern is to be multiplied by σ_{31}, which results in

1	C_{12}	1	C_{12}	1	C_{12}
C_{22}	C_{32}	C_{22}	C_{32}	C_{22}	C_{32}
1	C_{12}	1	C_{12}	1	C_{12}
C_{22}	C_{32}	C_{22}	C_{32}	C_{22}	C_{32}
1	C_{12}	1	C_{12}	1	C_{12}

Once we revert the route of the fleeing cock all dihedral symmetries into which the plane is decomposed are reflected by σ_{31}. Thereby all arrangements of the form

2	1	1	2
3	0	0	3

are turned over into

0	1	1	0
3	2	2	3

The sense of direction of each square is indeed reverted. But the situation involves a more comlex transition than reversion of the 4-cycle. Namely, it also involves a turnover from the vector plane (even part of $Cl_{2,0}$) to the complex plane (odd part). Again time reversion can be geometrized by algebraic manipulation, that is, multiplication of the pattern of orientation of the sand drawing by σ_{31}. This is a property of a large class of sand drawings.

Conflict in Mundane Time

There is a conflict between two sorts of time which goes back to the early middle ages and has survived until today, namely that between sacramental time and secular time. Modern physics perpetuates this conflict by distinguishing between *'mundane time'* and *'superlinear time'* (Raju 1994). We shall not go into superlinearity here as it cannot be in the least agreement with our present approach. But the concept of the mundane time requires some attention since it resembles some deeply rooted beliefs of most of us. Namely, we are convinced that the future is uncertain because it is unknown but that the past is certain even if it is unknown. This is a somewhat metaphysical belief. But it can show us something significant: The temporal order is depending on our knowledge and therefore on memory. The time concept is connected with the functions of thought and memory and therefore with our belief about what thought is and what memory is. Thought conceives of the mundane time as a tree of event sequences which is branching off towards the future (figure 13)

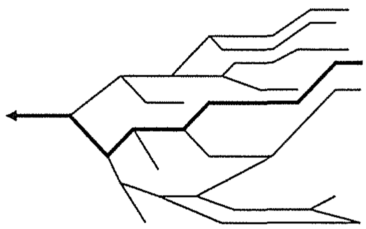


Figure 13: The Tree of Mundane Time

Note that the arrow is pointing towards the past. The thick line denotes a series of actual events while the other branches indicate possible events which, however, do not take place in reality.

Seen from the viewpoint of the present theory there is no such thing as a factual serial order of events in social awareness. Also the frame of physical time does not provide us with such an order. Different observers for example give different reports about one and the same time period with no definite chronology. It is interesting in this connection to ask why times are repeating though they are changing. Why is there a repetition of nationalism, ethnic conflicts, civil wars in almost any part of the world. Why are movements coming back? There were transcendentalists before World War 2 and there are associations of them now. There is a recurring belief in a golden age yet to come, throughout history it seems, and there is a periodic amplification of extreme rightists and bourgoise nationalists. Why is that so? Why is there a Rennaissance of facism? Is there a mechanism at work which we are perhaps not aware or even unable to become aware of? Consider the possibility that we might know more about the future than about the past. Now the past may turn out to be entirely uncertain because we do not know anything about it. Suppose we have forgotten what it was and that we have foregotten that we have forgotten. The past may have been discomforting and degrading. Or the past may comprise the information that we have murdered someone. So we have made considerable efforts to annihilate this information. On the other hand, we have contributed to a technosociety where the future is regarded as our own make. A transition from *certain past* and *unknown future* to *unknown past* and *certain future* requires two things, namely, (1) dislodging of experience as displacement of knowledge into the subconscious and (2) reversal of temporal order of experience. In this way, large parts of the discernible social fact become unaccessible. That is, the general social fact as disclosed in awareness decays into an accessible and an unaccessible part. The actors are deprived of their sight. As social change moves on such disorganization of temporal order can effect that the dislodged undiscernible part of social experience is projected onto the general social fact and becomes discernible through social practise. That is, the dislodged violence, the prohibited nationalism and facism which must not be realised in memory awareness and cognitive awareness are urged to become visible as pure social facts within the events of the discerned. Violence seizes its only chance and becomes visible to discursive consciousness by becoming real as social experience. Such a mechanism requires a reversal of the internal element of temporal order. Grüsser (1981) has described a mechanism of transposition of temporal serial order onto space arrangements of memory in proteins (RNS). The time series can be reverted by reverting the order of molecules in space. If temporal lattices come equipped with metastable patterns of biomolecules in space, than it is near at hand to conclude that we carry in us not only the future we plan but also the events we try to avoid most. It is the dislocated dying out experience which brings on its own recreation.

Note on Real Linear Time in Physics

In physics time reversal is possible and to some degree even necessary. Most classical equations of motion are symmetric with respect to a tilt in the arrow of time. In relativity space and time represent equivalent coordinates and are allowed to intermingle. But in quantum mechanics the two cannot be treated on an equitable footing. Instead, from the requirement that the negative energy operator H be bounded from below there follows the non-existence of a time operator with a spectrum $\sigma(T)$ contained in the real line R and at the same time satisfying the Heisenberg commutation relation $[H, T] = i\,\hbar$. This can be interpreted as an invalidity of the concept of linear time in quantum physics (Raju 1994, p. 143).

Reconstructing Physics

- Fourth Reconstruction -

Which Space-time?

The natural arena of physics is space-time. We have learned that, and students are still learning it today. We need not wonder at the meaning of that statement any more, and no one has to pose the question if the space-time is actually there or if it is constructed by thought, a formal, a mere algebraic or analytic reality. This is important to see because if we wish to take in a constructivist viewpoint the properties of the space-time are determined by the manifold of relevant models rather than by intuitive concepts such as dimension, signature, continuity, differentiability and the like. If it would turn out that the space-time cannot have a definite signature, that its dimension cannot be a natural number or that it is discontinuous and nowhere differentiable, we could no longer say what it is where physics is in. Its natural arena would turn out more mysterious than the events which are considered to take place in it. May be then we would have to ask if there is a corresponding natural arena for space-time. But the features of space-time are not only determined by our models and instruments — relativity, quantum theory, Clifford algebra, quantum deformation, renormalization, superstrings, non standard quantum covariant differentiation and so on — but also by the scientific communities who decide upon their validity. Good models may indeed be preferred to bad ones. But sometimes the good ones may be forgotten and the bad ones chosen. Then it can become very arduous to revive the old Ansatz or to regenerate some lost configurations.

The nowaday scientific community takes no definite standpoint as to which geometric structure is to be regarded fundamental for the theory. But there is a partition into different belief systems. Some consider the local differential geometry together with the Newman-Penrose formalism as their basic instrument of space-time analysis. Others lay stress on a thorough investigation of the geometric algebras. The field theoretical branch is more busy with superstrings. Still others who also do not take coordinate systems for granted turn over to fractal spaces. They invent scale-relativity, new forms of covariate derivatives and other novelties. It is not easy to decide precisely which alternative is the best or even only what is common ground for them all.

Reading a Cambridge Monograph on general relativity one came upon the suggestive statement that the "simplest example of a curved space is the surface of a sphere S^2 such as the surface of the earth" (Stewart 1993). For a reader in relativity, cosmology or astrophysics this may sound evident indeed but for those engaged with fractal geometry or chaos theory it is simply meaningless. The surface of the earth is by far the most complicated example of an irregular fractal space. We may only remember Bachelier's calculations about the lenght of the coast of Brittain. The physical examples of surfaces tend to disclose unbounded areas as soon as the resolution length approaches zero. Looked at the *'differentiable space-time manifolds'* from the superstring viewpoint the coordinates themselves are quantum fields so that the quantum fluctuations of those fields deconstruct the local diffeomorphism between space-time and the $\mathbf{R}^n$ thereby destroying the smooth character of the manifold.

This means that the property of space-time being a smooth manifold is true only in an approximate way and as long as one integrates out all very short distance (high frequency) modes. At a functional level the dimension of space-time is not even defined in general because there is no sense in which even locally it is diffeomorphic to $\mathbf{R}^n$. We may indeed ask if such integrating out can at all be understood as an integration in the exact sense. What could be the meaning, both physical and mathematical, to approximate a fractal space time by a differentiable manifold? Such approximation replaces a unit by a diverging measure and vice versa. The situation becomes even worse once we are forced to realise that there is no total concept of a stable manifold whether fractal or continuous.

Some researchers into quantum fractal dynamics emphasize that the notions of lenght η, surface η^2 and volume η^3 have only a hypothetical meaning. Therefore they begin with Mandelbrot's fundamental equation of fractal geometry:

[1] $N.\eta^{\Delta} = \text{constant with } 0 < \Delta < 2$

here N denotes the number of modes necessary to pin down the entire object at the resolution scale η. Only in cases where $\Delta = 1$, the term $N.\eta$ expresses the lenght ℓ of a Euclidean trajectory, but as $\Delta > 1$, we observe that $\ell = N.\eta \simeq \eta^{1-\Delta} \to \infty$, a diverging lenght. There are many indications that quantum mechanical trajectories have a dimension 2 rather than 1. One such fractal approach has been employed by Abbot and Wise (1981). Another indication is traced back to Feynman (1965, Schweber 1986) who knew that at a time-scale δt, the mean quadratic velocity of an electron is

[2] $\langle v^2 \rangle \propto \delta t^{-1}$.

Assuming that its quantum path has a fractal dimension <D, one should expect a relation between the space- and time-resolution of the form

[3] $\langle v^2 \rangle \cong (\delta x/\delta t)^2 \propto \delta t^{2[(1/D)-1]}$

which ultimately has to lead to D = 2, when one compares the exponent with Feynman's formula [2]. This calculation can be found in one of the fundamental papers on fractal space-time (Nottale 1993), where the above works of Feynman and Schweber are quoted. The fact that a quantum trajectory cannot possibly be a one-dimensional differentiable submanifold of the Minkowski space-time has been known to Heisenberg, Schrödinger and Born. In his fundamental paper on the kinematics of non-classical mechanics Heisenberg proposed to cancel out the geometric concept of a particle trajectory. He doubted that continuity and differentiability were meaningful notions at all in quantum mechanics. Even Einstein contemplated the renunciation of differentiability, but only in the microphysical domain where he believed the strong principle of equivalence had to hold true in any case. Only Max Born, it seems, held a critical position towards both differentiable trajectories and relativity. He denied "the possibility to observe 4-dimensional geodesics inside atomic dimensions" (Born 1939).

We are used to consider space-time as 3-, 4- or n-dimensional vector space $\mathbf{R}^n$ over the real number field. This may seem intuitively appealing and we have many different reasons why we are doing so. But the decision upon a definite value of n is critical. It depends on the physical process under observation. It may seem evident that macrophysical space has dimension three as it allows for three orthonormal basis vectors and equation [1] often brings on the solution $\Delta = 1$. This argument can be considered Galilean, Newtonian or even somewhat naiv but many of our more sophisticated arguments are not much better. Let S denote our space-time. Suppose, for the present, it is a vector space over some field and has a non-degenerate quadratic form, say

[4] $$Q(\mathbf{x}) = x_1^2 + x_2^2 + \ldots + x_p^2 - x_{p+1}^2 - \ldots - x_{p+q}^2 \quad \text{with } n = p + q.$$

Then our decision upon the design of the quadratic form of S is at the same time a definite decision on dim(S) = n. Lorentz invariance was a good reason why we chose n to be 4 and the Minkowski space **S** with signature (+, –, –, –) as a representant of the *'real'* physical space-time. Here we use the *'real'* in the classical greek sense meaning the domain of 'Wirkung' (cause and effect). But the word is critical. Because in the sense of 'real numbered' it cannot intuitively meet the realness because the geometric Clifford algebra $Cl_{1,3}$ of the space-time $\mathbf{R}^{1,3}$ is isomorphic to Mat(2, **H**), where **H** denotes the quaternion field. It is only in the inverse signature (+, +, +, –) that we are led to a matrix algebra Mat(4, **R**) over the real field. But we are allowed to go much further. We can argue that the physical laws of motion, which have become so famous, namely, the Maxwell- and the Dirac-equations cannot only be formulated in $Cl_{1,3}$ or $Cl_{3,1}$ but even in $Cl_{3,0}$, which means that the Clifford algebra of Euclidean 3-space is enough to embed the Dirac-equation. Or we can extend the dimension and formulate Dirac-Fueter equations in higher spaces $Cl_{p,q}$ of any dimension. Then what has been called the *'natural arena of physics'* is the geometric Clifford algebra generated by the vector space S over some field (**R**, **C** or **H**) rather than the space S itself. I think this is a very important aspect which has to do with the wholeness of our equations of motion.

Now our theories may become more sophisticated, conclusions more powerful, argumentation lines better organized and so forth. But still we have to accept that the dimension of the fundamental space S is a positive integer derived from some quadratic form, whether definite or indefinite, anyhow, from a sum of squares. But this is important. Since forms of squares have already been known to the old Greeks. They provide us with many curious identities. One of the most ancient is the following

[5] $(X_1^2+X_2^2)(Y_1^2+Y_2^2) = (X_1Y_1-X_2Y_2)^2+(X_1Y_2+X_2Y_1)^2$

It tells us that a product of two sums of two squares is itself a sum of two squares. Another is an identity known to Euler in 1770 which he used to prove Lagrange's theorem that every positive integer is a sum of four squares:

[6] $(X_1^2+X_2^2+X_3^2+X_4^2)(Y_1^2+Y_2^2+Y_3^2+Y_4^2) = (Z_1^2+Z_2^2+Z_3^2+Z_4^2)$

Equation [5] represents the fact that the norm of the product of two complex numbers Z1, Z2 is the product of their norms:

[5'] $|Z_1Z_2|^2 = |Z_1|^2|Z_2|^2$ with $Z_1= X_1 + iX_2$ and $Z_2= Y_1 + iY_2$

Equation [6] represents the analogous relation for quaternions as were discovered by William Hamilton (1853) in 1843. Briefly after Hamilton's introduction of the quaternions, Arthur Cayley discovered the octonions in 1845. They obey the following norm-identity:

[7] $(X_1^2+X_2^2+...+X_8^2)(Y_1^2+Y_2^2+...+Y_8^2) = (Z_1^2+Z_2^2+...+Z_8^2)$

This equation defines another field of numbers, namely, the octonions **O**. They form a non-associative division-ring. We are familiar with non-associative algebras because division with 'ordinary' (integer, real) numbers is not associative, for example $80\div(8\div2) \neq(80 \div8)\div2$. It was only in 1898 that for bilinear functions Z_k of X_i and Y_j Hurwitz worked out a positive decision on the dimensionality of all possible normed algebras over the real field:

Theorem: (Hurwitz 1898)

Let **K** be a field with char **K**≠2. The only values of n for which an identity of the type

$(X_1^2+X_2^2+...+X_n^2)(Y_1^2+Y_2^2+...+Y_n^2) = (Z_1^2+Z_2^2+...+Z_n^2)$

can hold, where the *Z_k are bilinear functions* of the X_i, Y_j with coefficients in **K** are the integers n = 1, 2, 4, 8. Hurwitz proved this for the complex number field **C**. But his proof could easily be generalized to any field with character unequal 2. It is because of two reasons that Hurwitz's proof gains a very special importance. The first is in its value for the classification of geometric algebras, the second in its extension to bilinear forms of the Y_j with coefficients in the polynomial ring $K(X_1+X_2+...+X_n)$. Considering the first, we find that **R**, **C**, **H**, **O** having dimension 1, 2, 4, 8 are the only normed algebras over the real numbers. The fields **R**, **C**, **H** are sufficient to represent any Clifford algebra $Cl_{p,q}$. Namely, each geometric algebra $Cl_{p,q}$ with (p–q) mod 4≠1 is isomorphic with a full matrix algebra over the division ring **R**, **C** or **H.** Now, it is usual in one of those algebras that the laws of physics are formulated, and scientists all over the world have invented numerous alternatives to do so, some of them strange or odd, others very beautiful. To give some highly interesting examples: When Baylis formulated some version of the relativistic theory of electrons, photons and Dirac spinors he did that in the Pauli algebra, that is, in $Cl_{3,0}$ (Baylis 1993). This example is not to be underrated. One might expect that the Pauli algebra representing the Clifford algebra of Euclidean 3-space can only provide non-relativistic spin matrices and is therefore too small to lodge the whole relativistic quantum dynamics of the Dirac-equation, which requires a complex valued Minkowski space-time. But the Minkowski space is a subspace of $Cl_{3,0}$.

When Gibbons (1993) revived some basic considerations by Eddington (1935, 1936) and investigated the Kummer configuration in relation to the geometry of Majorana spinors he used a set of projective geometries and Clifford algebras. When Parra proved that the Dirac equation gives rise to an inertial system of reference of its own and intrinsic to the equation he could do that within the framework of $Cl_{3,1}$. When the eight copies of the symmetric unitary group **SU**(3) were derived from the orientation symmetry of the Minkowski space-time again the Clifford algebra Ansatz was extremely useful. Namely, I could transpose the Gell-Mann Nishijima relation for the calculation of electric charges into pure geometric terms by building up basic reflections which generate space units together with color–triads in $Cl_{3,1}$. This makes it possible to explain the events of strong interaction as geometric properties and vice versa space as a property emerging through the interactions. There are many important approaches where equations of motion are being formulated in the geometric algebra rather than in the underlying vector space. Reformulations of Dirac-systems, calculations of mass spectra, twistors and space-time quantizations are often designed within the framework of geometric Clifford algebra But it is still only a minority of scientists who are able to see the significance of this powerful instrument of mathematics.

There is a certain tendency to use real representations, real Pauli matrices, real Dirac matrices or the so called space-time algebra (STA) which is also a matrix algebra with real entries. What are the reasons for such a move? At least one of them points into the direction of the extension of Hurwitz-1898 towards the theory of Pfister forms (multiplicative forms), the Artin-Schreier theory of the *formally real fields* and some new theorems on *extension fields.* To be able to go into this let us first define some approved predicates of what is *'order'* and *'formally real'*.

A field **K** is said to be ordered if a relation > is defined on **K** which satisfies

(1) If a, b $\in$ **K**, then either a > b or a = b or b > a
(2) If a > b, c $\in$ **K**, c > 0, then ac > bc
(3) If a > b, c $\in$ **K**, then a+c > b+c and further

The field **K** is formally real if –1 (or equivalently 0) cannot be expressed as a sum of squares in **K**. This is corresponding with the basic property of the reals that the only relations of the form $\Sigma_i a_i^2 = 0$ (with $a_i \in$ **R**) are trivial ones: $0^2 + 0^2 + \ldots + 0^2 = 0$. Then the following theorem has to hold:

Th.: Let **K** be an orderer field with respect to a fixed order in **K**, such that
(1) Positive elements have square roots in **K**
(2) Any polynomial of odd degree $\in$ R[X] has a root in **K**. Then $\sqrt{-1} \notin$ **K** and K($\sqrt{-1}$) is algebraically closed.

Suppose $\alpha \in$ **K** and $-1 = \alpha^2$ then **K** is not formally real and thus not ordered. But it is given to be ordered, therefore $\sqrt{-1}$ is not in **K**.

This theorem represents a generalization to the observation that the complex field is not ordered but any formally real field is ordered. Observing the basic fields **R**, **C**, **H**, **O** it is striking how the axioms of the ordered field **R** are lost one after the other while the dimension of the division ring is raised. **C** is a field commutative and associative under multiplication but it does not have an order relation. The quaternions **H** form an associative division ring only, while **O** is not even associative. In the end associativity, commutativity and order are lost. Thus a representation of any geometric algebra by matrix algebras over the real field preserves an order relation. May be that this is the reason why some of us prefer the STA $\equiv$ $Cl_{4,0}$ with its matrix representation Mat(4, **R**) to $Cl_{1,3}$ which is Mat(2, **H**). By the same reason the $Cl_{3,1}$ may be preferred to $Cl_{1,3}$ though both represent Clifford algebras of a Minkowski space-time up to a reversion of signature. We have $Cl_{3,1} \simeq$ Mat(4, **R**) and $Cl_{1,3} \simeq$ Mat(2, **H**) and the real field is commutative and ordered while the quaternion algebra is neither commutative nor ordered. But we must not overlook that a real Clifford algebra such as $Cl_{3,1}$ nevertheless contains many square roots of –1 with both geometric and physical meaning.

To come to a definite decision on the concept of space time is not easy and by the time has turned into a question of sociology and linguistics rather than of physics. One has to understand the grammar of model-building and the general fact of social exchange not only of knowledge but also of power. That is, the best model has no value if nobody defends it, and even all the power of a scientific community cannot legitimate a wrong model. Often it is difficult to take a choice between two almost equivalent frameworks. Take spinors as a prominent example. As spinors of a Minkowskian space-time-algebra belong to a minimal left ideal multivector subbundle either of $Cl_{3,1}$ or alternatively of $Cl_{1,3}$ their existence conditions are not the same in both algebras. But it has been pointed out that they are stronger in $Cl_{3,1}$ than in $Cl_{1,3}$. Therefore we must be aware that a decision in favour of one of those seemingly equally justified signatures of the local metric implies physical consequences. Such is no easy choice and it can be resulting from the power of actors rather than from observations and reasonable thought.

The above considerations are very important for a space-time with integer dimension n=4. But how is one to proceed in case it turns out that space-time has a fractal dimension of, say, 3.98? Such a possibility has been contemplated by El Naschie (1995). What about the Clifford algebra $Cl_{3.98,1.5}$? Some scientists say that there arises a barrier which renders obsolete the use of the geometric algebra: "The vectorial and differential formalism along with their associated algebra (duality principle: internal and external algebra), create a technical barrier when applied by extension to a fractal geometry. Given its level of inefficiency, this barrier cannot be overcome with physical artefacts". (Le Méhauté et al. 1995) It is true that a mathematical problem cannot be overcome with artefacts. But it can sometimes be solved in a formal way. Fractals are indeed compatible with geometric algebras and it is even possible to define nondifferentiable submanifolds of space-time with a fractal dimension within the Clifford algebra formalism. Also it is possible to define a set of so-called non-standard real numbers. (Stroyan and Luxemburg 1976) It can be shown that such numbers are unique sums of one standard real and one infinitesimal number

[8] $\quad {}^*\mathbf{R} = \{\xi \,/\, \xi = x+dx \text{ with } x\in\mathbf{R}\}$

By defining an appropriate order relation on *$\mathbf{R}$ we can extend the axioms of order from the real to the non standard real field. We have to put

[9] $\quad x > dx$ for any $x>0$ and

[10] $\quad dx > (dx)^{\alpha}$ for $\alpha\in\mathbf{R} \wedge \alpha > 1$ and so on.

Then by a well known statement of the Artin-Schreier theory we can conclude from the unique order on the non standard real field *$\mathbf{R}$ that it is formally real and real closed. That is, no proper extension of *$\mathbf{R}$ is formally real. On this basis one can try to construct an associative algebra *Cl(*R)* over the vector space *$\mathbf{R}$, which obeys the desired demands of geometric algebras and comprises nondifferentiable manifolds. But I shall not go into this here, because there is a more direct approach which allows for the definition of fractal trajectories within the well-known theory of Clifford algebra over the real vector space $\mathbf{R}$. This has the advantage that some of our significant results about the relations between spin, charge, mass and some other quantum numbers can be built into the theoretic design.

Three Fundamental Questions

Among the many questions which arise in the midst of conflicting theories and schools of theoretical physics there are three which to me seem of preeminent significance. The first is philosophic and has already been posed by Whitehead:

Q1 How is cognition related to nature? What does it imply that nature is disclosed to mind in awareness though physics has concluded that it need not be concerned with sense-awareness? The other two are indeed questions of physics, namely

Q2 How is matter connected with space-time and why are they considered separate at all? How are the symmetry properties of physical interaction related to the symmetries of space-time. Why Is there a partition into inner versus outer symmetries?

Q3 Is space-time continuous or fractal and is there a stable template of space-time at all? Are there equations of motion such as for instance the Einstein equations? Suppose space-time behaves like a fractal, should we conceive of it as static or dynamic? Does it relate to sense awareness or does it not relate to it?

It is impossibe to settle those questions or to give an obliging answer within the present day framework of mathematical physics. But I can at least show the way how the right answers might be found. For convenience I begin with the second question. Namely, we start from the following hypothesis:

H1: Space-time and matter are not essentially different. Chisholm (1993) has posited this statement in a simple and elegant form: "There is no distinction between space-time and the internal interaction-space". This is so to say our innermost credo.

But we go a little step further and postulate that both material field and units of space are created by what I call a *reflection field.* Namely, the fundamental field is both reflected and reflecting. It is in a permanent state of self-interaction. This procedure will allow us to work out a theory where inner and outer symmetries mutually imply each other. Let us go into medias res and choose as a first framework the geometric Clifford algebra. We use it as a natural arena of the Dirac field where we can set up its well-known Dirac equation. Then we proceed by locating the interaction symmetries **U**(1), **SU**(2) and **SU**(3) one after the other in such a way that Euclidean 3-space together with its orientation and the interactive properties of fields are generated.

The Dirac Equation

Consider the Minkowski space in the opposite metric $\mathbf{R}^{3,1}$ with its Clifford algebra $Cl_{3,1} \simeq$ Mat(4,**R**). The Dirac equation in this space is

[11] $\partial\psi + \mathrm{i}\, \mathrm{e}\, \mathbf{A}\psi = \mathrm{m}\psi$

This form guarantees that for a real particle with the unit velocity

[12] $u = u_1\mathbf{e}^1 + u_2\mathbf{e}^2\, u_3\mathbf{e}^3 + u_4\mathbf{e}^4$

the square u^2 is real and time-like: $u_4^2 > u_1^2 + u_2^2 + u_3^2$. It is possible to represent $Cl_{3,1}$ either by real matrices (Lounesto 1996, p. 15) or in a complexified structure given by C ⊗ Mat(4,R) $\simeq$ Mat(4,C). One possible complex representation is based on the following unit vectors (Schmeikal 1996):

[13]

$$\mathbf{e}_1 = \begin{bmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{bmatrix} \quad \mathbf{e}_2 = \begin{bmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{bmatrix} \mathbf{e}_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \mathbf{e}_4 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

where σ_1, σ_2 are Pauli matrices and $i = \sqrt{-1}$. This representation is obtained by the following choice of the primitive idempotent

[14] $f_1 = (1/2)(1 + \mathbf{e}_1)\,(1 + \mathbf{e}_{34})$

Together with

$$f_2 = (1/2)(1 - \mathbf{e}_1)\,(1 + \mathbf{e}_{34})$$
$$f_3 = (1/2)(1 - \mathbf{e}_1)\,(1 - \mathbf{e}_{34})$$
$$f_4 = (1/2)(1 + \mathbf{e}_1)\,(1 - \mathbf{e}_{34})$$

we have a maximal set $F_1 = \{f_i\}$ of mutually annihilating primitive idempotents which sum up to unity. These are used in section 8 to built up the basic reflections from which we calculate subspaces invariant under ***SU(3)***.

Locating the Symmetries *U(1)* and *SU(2)*

The special unitary group ***SU(2)*** $\simeq Cl_{3,0}$ can be located by omitting the unit $\mathbf{e}_4$ from the basis of $Cl_{3,1}$. It can further be located within the copies of ***SU(3)*** $\subset Cl_{3,1}$ which will be described in a later section. The Pauli algebra $Cl_{3,0}$ contains central invertible elements having the form $x + y\,\mathbf{e}_{123} \in Cl^{-}_{3,0}$ with $x, y \in R$. A subset of these forms a group

[15] $W_1 = \{ x + y\,\mathbf{e}_{123} \mid x, y \in R;\ x^2 + y^2 = 1\} \simeq U(1)$

isomorphic to the unitary group $U(1)$. Being aware that the expressions of the form $(1+ \mathbf{e}_i)$, $(1+ \mathbf{e}_{i4})$, $(1+ \mathbf{e}_{ik4})$ (for i, k $\neq$ 4) are not invertible, there remain five more groups of the above type

[16]
$$W_2 = \{ x + y\,\mathbf{e}_{12} \mid x, y \in R;\ x^2 + y^2 = 1\} \simeq U(1)$$
$$W_3 = \{ x + y\,\mathbf{e}_{13} \mid x, y \in R;\ x^2 + y^2 = 1\} \simeq U(1)$$
$$W_4 = \{ x + y\,\mathbf{e}_{23} \mid x, y \in R;\ x^2 + y^2 = 1\} \simeq U(1)$$
$$W_5 = \{ x + y\,\mathbf{e}_{4} \mid x, y \in R;\ x^2 + y^2 = 1\} \simeq U(1)$$
$$W_6 = \{ x + y\,\mathbf{e}_{1234} \mid x, y \in R;\ x^2 + y^2 = 1\} \simeq U(1)$$

The Field of Reflections

Consider the quaternions as have been discovered by Hamilton together with the four primitive idempotents as were chosen in section 3. Next form the following reflections which somehow resemble the reflection operators 'σ' which have been used so successfully in crystallography by Schönfließ.

Consider reflections

[17]
$$\begin{array}{ll}
s_{11} = f_1 + f_2 + \mathbf{e}_{12}(f_4 - f_3) & s_{12} = f_1 + f_2 + \mathbf{e}_{12}(f_3 - f_4) \\
s_{21} = f_1 + f_3 + \mathbf{e}_{13}(f_2 - f_4) & s_{22} = f_1 + f_3 + \mathbf{e}_{13}(f_4 - f_2) \\
s_{31} = f_1 + f_4 + \mathbf{e}_{23}(f_3 - f_2) & s_{32} = f_1 + f_4 + \mathbf{e}_{23}(f_2 - f_3) \\
s_{41} = f_2 + f_3 + \mathbf{e}_{23}(f_1 - f_4) & s_{42} = f_2 + f_3 + \mathbf{e}_{23}(f_4 - f_1) \\
s_{51} = f_2 + f_4 + \mathbf{e}_{13}(f_3 - f_1) & s_{52} = f_2 + f_4 + \mathbf{e}_{13}(f_1 - f_3) \\
s_{61} = f_3 + f_4 + \mathbf{e}_{12}(f_2 - f_1) & s_{62} = f_3 + f_4 + \mathbf{e}_{12}(f_1 - f_2)
\end{array}$$

$$\begin{array}{ll}
s_{71} = g_1 + g_2 + \mathbf{e}_{12}(g_4 - g_3) & s_{72} = g_1 + g_2 + \mathbf{e}_{12}(g_3 - g_4) \\
s_{81} = g_1 + g_3 + \mathbf{e}_{13}(g_2 - g_4) & s_{82} = g_1 + g_3 + \mathbf{e}_{13}(g_4 - g_2) \\
s_{91} = g_1 + g_4 + \mathbf{e}_{23}(g_3 - g_2) & s_{92} = g_1 + g_4 + \mathbf{e}_{23}(g_2 - g_3) \\
s_{101} = g_2 + g_3 + \mathbf{e}_{23}(g_1 - g_4) & s_{102} = g_2 + g_3 + \mathbf{e}_{23}(g_4 - g_1) \\
s_{111} = g_2 + g_4 + \mathbf{e}_{13}(g_3 - g_1) & s_{112} = g_2 + g_4 + \mathbf{e}_{13}(g_1 - g_3) \\
s_{121} = g_3 + g_4 + \mathbf{e}_{12}(g_2 - g_1) & s_{122} = g_3 + g_4 + \mathbf{e}_{12}(g_1 - g_2)
\end{array}$$

where the g_i are obtained from the f_i by regarding the isomorphism $Cl_{3,0} \simeq Cl^{+}_{3,1}$ determined by the correspondences $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \simeq \mathbf{e}_{14}, \mathbf{e}_{24}, \mathbf{e}_{34}$. They can also be calculated by a unitary transformation

[18] $u_2 = (1/\sqrt{2})(1 - \mathbf{e}_4) \in W_5$.

For consider the set $G_1 = \{g_i\}$; we have

[19] $G_1 = u_2 F_1 (u_2)^{\sim}$ where $(u_2)^{\sim}$ is the Clifford conjugate to u_2

Generating Space

The 24 reflections $s_{j\alpha}$, with j = 1, 2, ..., 12 and α = 1, 2 are close relatives of the Schönfließ symbols σ_d', σ_d'' (Belger & Ehrenberg 1981, Schmeikal 1996). They have extremely interesting properties: they are invertible and their squares equal unity

[20] $\forall\, j,\alpha \quad (s_{j\alpha})^2 = +1$

By reversion $^{\sim}$ they are normalized to unit vectors in the basis of $\mathbf{R}^3$. Precisely, they satisfy the 24 equations

[21]
$$\begin{array}{llll}
s_{11}\,(s_{11})^{\sim} = -\mathbf{e}_2; & s_{12}\,(s_{12})^{\sim} = +\mathbf{e}_2; & s_{21}\,(s_{21})^{\sim} = +\mathbf{e}_3; & s_{22}\,(s_{22})^{\sim} = -\mathbf{e}_3 \\
s_{31}\,(s_{31})^{\sim} = +\mathbf{e}_1; & s_{32}\,(s_{32})^{\sim} = +\mathbf{e}_1; & s_{41}\,(s_{41})^{\sim} = -\mathbf{e}_1; & s_{42}\,(s_{42})^{\sim} = -\mathbf{e}_1 \\
s_{51}\,(s_{51})^{\sim} = +\mathbf{e}_3; & s_{52}\,(s_{52})^{\sim} = -\mathbf{e}_3; & s_{61}\,(s_{61})^{\sim} = +\mathbf{e}_2; & s_{62}\,(s_{62})^{\sim} = -\mathbf{e}_2 \\
s_{71}\,(s_{71})^{\sim} = -\mathbf{e}_3; & s_{72}\,(s_{72})^{\sim} = -\mathbf{e}_3; & s_{81}\,(s_{81})^{\sim} = -\mathbf{e}_1; & s_{82}\,(s_{82})^{\sim} = +\mathbf{e}_1 \\
s_{91}\,(s_{91})^{\sim} = +\mathbf{e}_2; & s_{92}\,(s_{92})^{\sim} = -\mathbf{e}_2; & s_{101}\,(s_{101})^{\sim} = -\mathbf{e}_2; & s_{102}\,(s_{102})^{\sim} = +\mathbf{e}_2 \\
s_{111}\,(s_{111})^{\sim} = +\mathbf{e}_1; & s_{112}\,(s_{112})^{\sim} = +\mathbf{e}_1; & s_{121}\,(s_{121})^{\sim} = +\mathbf{e}_3; & s_{122}\,(s_{122})^{\sim} = +\mathbf{e}_3
\end{array}$$

It is on account of these normalization equations that we shall say that space is brought forth by a generative process of strong interaction. For the present we have to notice that the $s_{j\alpha}$ cannot belong to Pin(3,1) because in that case, by reversion they would be normalized to unity ± 1, but not to unit vectors. They are composed by both even and odd components. But they do not belong to the Lipschitz group $\Gamma_{3,1}$ because otherwise vectors $\mathbf{x} \in \mathbf{R}^{3,1}$ would be transformed into vectors $s\,\mathbf{x}\,(s^\wedge)^{-1} = s\,\mathbf{x}\,s^\wedge \in \mathbf{R}^{3,1}$. That this is not the case can be seen from their transformative properties (table 1):

Table 1: Transformation properties of reflections $S_{j\alpha}$

	Products											
	$s_{j\alpha}\, e_i\, s_{j\alpha}{}^{\wedge}$				$s_{j\alpha}\, e_i\, s_{j\alpha}$				$s_{j\alpha}\, e_i\, s_{j\alpha}{}^{\sim}$			
	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
s_{11}	e_1	e_{234}	$-e_{23}$	$-e_{24}$	e_{134}	e_2	$-e_{24}$	$-e_{23}$	$-j$	-1	$-e_4$	$-e_3$
s_{12}	e_1	e_{234}	e_{23}	e_{24}	e_{134}	e_2	e_{24}	e_{23}	j	1	$-e_4$	$-e_3$
s_{21}	$-e_{13}$	e_2	$-e_{134}$	e_{34}	e_{34}	$-e_{124}$	e_3	$-e_{13}$	$-e_4$	j	1	$-e_1$
s_{22}	e_{13}	e_2	e_{134}	$-e_{34}$	e_{34}	e_{124}	e_3	$-e_{13}$	e_4	j	-1	e_1
s_{31}	$-e_{124}$	e_{12}	e_3	e_{14}	e_1	$-e_{14}$	e_{234}	$-e_{12}$	1	e_4	$-j$	e_2
s_{32}	e_{124}	e_{12}	e_3	e_{14}	e_1	$-e_{14}$	$-e_{234}$	e_{12}	1	$-e_4$	j	$-e_2$
s_{41}	e_{124}	$-e_{12}$	e_3	$-e_{14}$	e_1	$-e_{14}$	$-e_{234}$	$-e_{12}$	-1	$-e_4$	$-j$	$-e_2$
s_{42}	$-e_{124}$	$-e_{12}$	e_3	$-e_{14}$	e_1	e_{14}	e_{234}	e_{12}	-1	e_4	j	e_2
s_{51}	$-e_{13}$	e_2	e_{134}	e_{34}	$-e_{34}$	e_{124}	e_3	e_{13}	e_4	$-j$	1	e_1
s_{52}	e_{13}	e_2	$-e_{134}$	$-e_{34}$	$-e_{34}$	$-e_{124}$	e_3	e_{13}	$-e_4$	$-j$	-1	$-e_1$
s_{61}	e_1	$-e_{234}$	e_{23}	e_{24}	$-e_{134}$	e_2	$-e_{24}$	$-e_{23}$	$-j$	1	e_4	e_3
s_{62}	e_1	$-e_{234}$	$-e_{23}$	$-e_{24}$	$-e_{134}$	e_2	e_{24}	e_{23}	j	-1	e_4	e_3

Another important feature of reflections $s_{j\alpha}$ is given by the multiplication rules

[22] $\quad s_{j1}\, s_{j2} = C_{j2} \quad$ and $\quad (C_{j2})^2 = 1$

Their meaning is well known in traditional geometry: in the same way the product of reflections σ_d', $\sigma_d'' \in \mathbf{D}_{2d}$ bring forth a half turn C_2 and thereby a nontrivial central element, and just as the diagonal reflections in the octahedral group generate half turns C_{j2}, so the reflections $s_{j\alpha}$ give us *Clifford orientation numbers* analogous to Schönfließ symbols C_{j2}. The index 2 indicates that the C_{j2} are generalized Schönfließ symbols for period 2. Before we go further, we have to anticipate that the f_i are ***SU(3)***- symmetric states (Greider and Weiderman 1988, Chisholm 1992, Schmeikal 1996). They are sometimes interpreted as a set of one lepton and three quarks. The action of reflections $s_{j\alpha}$ on the f_i are transpositions of the type

[23] $\quad s_{j\alpha}\, f_i\, s_{j\alpha} = f_k \quad$ and $\quad s_{j\alpha}\, f_k\, s_{j\alpha} = f_j$

For indices j = 1, 2, ..., 6 we have

[24] $\quad s_{j\alpha}\, F_1\, s_{j\alpha} = F_1 \quad$ and $\quad s_{j\alpha}\, G_1\, s_{j\alpha} = u_1\, F_1\, (u_1)^{-1}$

There is an equation analogous to [19], namely

[25] $\quad F_2 = u_1\, F_1\, (u_1)^{-1}$

where F_2 is another set of primitive idempotents (see Lounesto 1996 p. 15) derived from the expression $(1/2)(1 + \mathbf{e}_1)(1/2)(1 + \mathbf{e}_{24})$ and u_1 is the unitary transformation

[26] $\quad u_1 = (1/\sqrt{2})(1 + \mathbf{e}_{123}) \in W_1 \simeq U(1);$[27]

So the reflections $s_{j\alpha}$; j=1, 2, ..., 6, α=1,2; transform *SU(3)*-symmetric states into *SU(3)*-symmetric states up to some unitary transformation u. This gives freedom to physical interpretation.

Locating the Octahedral Orientation Symmetries

Procedure: In order to unveil the geometric origin on the **SU**(3)-symmetry in the geometric algebra of the Minkowski space $R^{3,1}$, we first have to locate in $Cl_{3,1}$ the octahedral orientation symmetries **O**. After that, the generators of each copy of *SU(3)* can be derived from **O** in a most natural way.

To represent any finite crystallographic subgroup of the connected component of $\mathbf{SO}^{+}(p,q) \subset \mathbf{SO}(p,q) \subset \mathbf{O}(p,q)$, we usually consider two-fold covers ("double-groups") in **Spin**(p,q). In this present case this would mean that we had to choose the even operators $S_{11} = (1/\sqrt{2})(\mathbf{e}_{12} + \mathbf{e}_{13})$ and $C_{14} = (1/\sqrt{2})(1 - \mathbf{e}_{13})$ [28] as a minimal basis for ${}_{\delta}\mathbf{O} \subset SU(2) \subset Cl_{3,1}$. However, I have not been successful in identifying the generators of *SU*(3) in this representation. Therefore I first attempted to write down a representation of **O** by Dirac matrices in Mat(4,C) and then pondered over the question how the basis thus obtained could be expressed in terms of reflections built up by the primitive idempotents in $Cl_{3,1}$. This was a rather arduous task. But it finally led to the location of eight copies of *SU*(3) by the support of the symbolic calculator **CLICAL**. Other copies following the same construction principle by using some further primitive idempotent have been excluded. Eight copies of *SU*(3) could be derivated from eight copies of the group **O** which contain the fundamental color operators T_{j3}. The eight representations of **O** by multivectors in the geometric algebra are not double covers. They can be generated by subsets of reflections from equations [7] in the following way:

[27]
$$\begin{array}{lll}
\mathbf{O}_1 \ldots s_{1\alpha}, s_{2\alpha}, s_{3\alpha}; & \mathbf{O}_2 \ldots s_{1\alpha}, s_{4\alpha}, s_{5\alpha} & \\
\mathbf{O}_3 \ldots s_{2\alpha}, s_{4\alpha}, s_{6\alpha}; & \mathbf{O}_4 \ldots s_{3\alpha}, s_{5\alpha}, s_{6\alpha} & \\
\mathbf{O}_5 \ldots s_{7\alpha}, s_{8\alpha}, s_{9\alpha}; & \mathbf{O}_6 \ldots s_{7\alpha}, s_{10\alpha}, s_{11\alpha} & \\
\mathbf{O}_7 \ldots s_{8\alpha}, s_{10\alpha}, s_{12\alpha}; & \mathbf{O}_8 \ldots s_{9\alpha}, s_{11\alpha}, s_{12\alpha} & \text{with } \alpha = 1, 2
\end{array}$$

[27] Notice, although both groups W_1 and W_5 are isomorphic with $U(1)$, their elements behave differently. The element $u_2 = (1/\sqrt{2})(1 - e_4) \in W_5$ is inverted by Clifford conjugation, that is, $(u_2)^{-1} = (u_2)^{-}$ whereas u_1 is inverted by reversion: $(u_1)^{-1} = (u_1)^{\sim}$.

[28] Notice, S_{11} is a reflection with period 2 in **SO**(3,1) but period 4 in $SU(2) \subset SL(2,C)$. Also the Schönfließ symbol C_{14} for rotations by $\pi/2$ in space are mapped onto period-8 spinors by that representation.

Minimal basis to each $\mathbf{O}_i$ is given by the first reflection and one spinor of the type c_{k4}. This is a period four spinor decomposed as a product of the first reflection, say, s_{r1} the second s_{r2} and s_{k1}. Consider for example the group $\mathbf{O}_1$.

A minimal generating basis is

[28] s_{11} and $c_{24} = s_{11}\, s_{12}\, s_{21}$

Orientation quantum numbers are

[29] $c_{12} = \mathbf{e}_{34}$ $c_{22} = \mathbf{e}_{134}$ $c_{32} = \mathbf{e}_1$

Period 4-spinors are

[30] $c_{14} = s_{31}\, s_{32}\, s_{11}$ $c_{24} = s_{11}\, s_{12}\, s_{21}$ $c_{34} = s_{21}\, s_{22}\, s_{31}$

$(c_{14})^{-1} = s_{11}\, s_{32}\, s_{31}$ $(c_{24})^{-1} = s_{21}\, s_{12}\, s_{11}$ $(c_{34})^{-1} = s_{31}\, s_{22}\, s_{21}$

Tetrahedral spinors of period 3 are

[31] $t_{13} = s_{11}\, s_{31}$ $t_{23} = s_{11}\, s_{22}$ $t_{33} = s_{12}\, s_{31}$ $t_{43} = s_{21}\, s_{32}$

$(t_{13})^{-1} = s_{31}\, s_{11}$; $(t_{23})^{-1} = s_{22}\, s_{11}$; $(t_{33})^{-1} = s_{31}\, s_{12}$; $(t_{43})^{-1} = s_{32}\, s_{21}$

Together with unity these are the 24 operators of an octahedral orientation symmetry.

Notice, the C_{j2} are nothing else than the components of our primitive idempotent f_1, which is $f_1 = (1/4)(1 + \mathbf{e}_1 + \mathbf{e}_{34} + \mathbf{e}_{134})$. Therefore we observe the identity

[32] $f_1 = (1/4)(1 + c_{12} + c_{22} + c_{32})$

Representing $\mathbf{O}_1$ in the Dirac algebra [section 3] its operators turn out to have a single 1 in the first rows and columns. This is due to the design of reflections s_{11} to s_{32} equal to $f_1 + \ldots$ Therefore, omitting the first row and column in each operator, we obtain a special unitary representation of $\mathbf{O}$ in Mat(3,$\mathbf{C}$). The c_{i2} are diagonal with entries ± 1.

Generating *SU*(3)

Next we have to calculate the multivectors.

[33] $\lambda_1 = (1/2)(s_{31} - s_{32})$ $\lambda_2 = (i/2)(c_{34} - (c_{34})^{-1})$

$\lambda_3 = (1/2)(c_{12} - c_{22})$ $\lambda_4 = (i/2)(c_{24} - (c_{24})^{-1})$

$\lambda_5 = (1/2)(s_{21} - s_{22})$ $\lambda_6 = (i/2)(c_{14} - (c_{14})^{-1})$

$\lambda_7 = (1/2)(s_{11} - s_{12})$ $\lambda_8 = (-1/\sqrt{12})(1 + 3c_{32})$

Representing the $\lambda_i \in Cl_{3,1}$ in the Dirac algebra chosen, these are translated into matrices of Mat(4,C). All of them have but zeros in the first rows and columns. But they are indeed nothing other then Gell-Mann matrices. We can reduce them to special unitary matrices in Mat(3,C) by simply omitting the first row and column. Consider for example the matrix $\lambda_5 = (1/2)(s_{21} - s_{22})$ in the basis [13]:

$$\lambda_5 = (1/2)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 1 & 0 \\ 0 & i & 0 & 0 \end{bmatrix} - (1/2)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 1 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix} =$$

$$= \left[\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{array}\right]$$

Casimir operators are

[34] $T_z = (1/4)(c_{12} - c_{22}) = (1/2)(f_2 - f_3)$ **isospin**
and $Y = -1/6 - c_{32}/2 = -1/6 - e_1/2$ **hypercharge.**

The **Gell-Mann-Nishijima relation** $Q = Y/2 + T_z$ in its geometric form is now

[35] $Q = f_1 - 1/3$ **electric charge.**

The operators c_{12}, c_{22}, c_{32} are Clifford numbers of orientation. They take values ±1 only for quarks, but are rational for hadrons and nucleons. For example the following table lists the invariants T_z, Y, Q, c_{12}, c_{22}, c_{32} for a neutron, proton, $\Lambda°$ and a red u-quark:

Table 2: Orientation numbers for p, n, $\Lambda°$ and u-red

	T_z	Y	Q	c_{12}	c_{22}	c_{32}
n	–½	1	0	–1/3	5/3	–7/3
p	½	1	1	5/3	–1/3	–7/3
$\Lambda°$	0	0	0	–1/3	–1/3	–1/3
u-red	½	1/3	2/3	+1	–1	–1

Color Tetrads

Represented in the Dirac algebra to basis [13] the color spinor $t_{13} \in \mathbf{O}$ is the matrix

$$t_{13} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -i \\ 0 & i & 0 & 0 \end{bmatrix}$$

it transforms the idempotents according to a color rotation $f_2 \to f_3 \to f_4 \to f_2$, but does not alter f_1. It induces a color rotation also within the set of orientation operators c_{i2}. If we calculate the expressions $t_{13}\, c_{i2}\, (t_{13})^{-1}$ we observe a sequence $c_{12} \to c_{22} \to c_{32} \to c_{12}$. Thus, while the *SU*(3)-symmetric states are transformed into one another, their *quantum numbers of orientation* are altered accordingly. But that means that the components $\mathbf{e}_{34}$, $\mathbf{e}_{134}$, $\mathbf{e}_1$ of the primitive idempotents f are rotated accordingly too, that is, we have

[36] $\quad t_{13}\, \mathbf{e}_{34}\, (t_{13})^{-1} = \mathbf{e}_{134}$

$$t_{13}\, \mathbf{e}_{134}\, (t_{13})^{-1} = \mathbf{e}_1$$

$$t_{13}\, \mathbf{e}_1\, (t_{13})^{-1} = \mathbf{e}_{34}$$

Recall, we have located eight copies of the octahedral orientation symmetry giving rise to eight copies of the $SU(3)$. So it is interesting to identify the subspaces of the Clifford algebra which possess the associated orientation symmetry **O** of the Euclidean space. This is not at all a trivial question, and the answer is: the subspace which represents the action of $\mathbf{O}_1$ in the correct multiplicative order is generated by $\{\mathbf{e}_2, \mathbf{e}_{14}\}$. It is only in this subspace of $Cl_{3,1}$ where a proper rotation c_{14} of period 4 actually generates a rotation among four states. In the subspace generated by $\{\mathbf{e}_1, \mathbf{e}_{34}\}$ it collapses into two states. We have for example

[37] $$c_{14}\, \mathbf{e}_1\, (c_{14})^{-1} = \mathbf{e}_{134}$$

$$c_{14}\, \mathbf{e}_{134}\, (c_{14})^{-1} = \mathbf{e}_1$$

which are transpositions between two units. This is not so in $\{\mathbf{e}_2, \mathbf{e}_{14}\}$.

Let us be aware, since almost thirty years it was known that a necessary condition for the existence of spinor fields is that the space-time be orientable (Geroch 1968). But at no time has there been any complete concept of orientation. Nor has anybody found similar criteria for isospinor fields. The reason is in the lacking of a sufficiently complete and consistent methodology. Such methodology is most naturally introduced by the Clifford algebra formalism.

The set $\{1, \mathbf{e}_2, \mathbf{e}_{14}, \mathbf{e}_{124}\}$ forms an oriented tetrad of units in the Clifford algebra $Cl_{3,1}$. It will be called a *color tetrad* of the Minkowski space-time. For consider the operator $t_{23} = s_{11}s_{22}$. It induces a color rotation in the basis

$$\begin{tikzcd}
 & \mathbf{e}_2 \arrow[dr] & \\
\mathbf{e}_{124} \arrow[ur] & & \mathbf{e}_{14} \arrow[ll]
\end{tikzcd}$$

There is a set of 8 unit vectors $(½)(\pm 1 \pm \mathbf{e}_2 \pm \mathbf{e}_{14} \pm \mathbf{e}_{124})$ forming a color unit cube on which the action of orietation symmetries can be tested.

Before I go deeper into questions of orientation and disorientation of fields, I would like to discuss some fundamental problems of fractal dimension as we said that particle trajectories as are actually posited in observation have dimension two rather than 1. There are two possiblities to go on. The first consists in constructing an associative algebra $Cl(^*R)$ over the vector space *R of formally real numbers which obeys the desired demands of geometric algebras and comprises nondifferentiable manifolds. The second is more fundamental and begins with the construction of a spin fractal in an ordinary Clifford algebra. For what follows I have chosen the second procedure. It will require only basic knowledge about spinor spaces but demonstrate the idea.

Fractals in Spinor Space

To obtain a first impression of the argument consider the Pauli algebra $Cl_{3,0}$ represented by the matrix algebra Mat(2, **C**) over the complex field. Let $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ be the orthonormal basis vectors of euclidean $\mathbf{R}^3$. Take the primitive idempotent $f_1 = ½\,(1+\mathbf{e}_3)$ in $Cl_{3,0}$ which generates the minimal left ideal $S = Cl_{3,0}\, f_1$. For any $\psi \in S$ and $\alpha \in Cl_{3,0}$ also we have $\alpha\psi \in S$. The division ring $\mathbf{K} = f_1 Cl_{3,0}\, f_1 \simeq \mathbf{C}$ has the basis

[38] $f_1 = ½\,(1+\mathbf{e}_3) \qquad f_i = ½\,(\mathbf{e}_{12}+\mathbf{e}_{123})$

with $f_i^2 = f_1$ which is the reason why **K** is isomorphic with the complex field. The mapping S x **K** $\to$ S equivalent to $(\psi, \alpha) \to \psi\alpha$ makes S a right-sided **K**-linear space. Provided with this linear structure S becomes a spinor-space and its elements are called spinors. As a **K**-linear space S has dimension 2 and basis

[39] $f_1 = ½\,(1+\mathbf{e}_3) \qquad\qquad f_i = ½\,(\mathbf{e}_1+\mathbf{e}_{13})$

and it is in this basis that the unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ of the euclidean $\mathbf{R}^3$ have the prominent matrix representations

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

These matrices from Mat(2, **C**) are denoted as Pauli matrices and matrices with a vanishing second column are called Pauli spinors. These results are well known today but they were not known then. According to a hypothesis first stated by Uhlenbeck and Goudsmit (1925) there existed a magnetic moment $\mu(e^-)$ of the electron which could not be thought of as originating from its trajectorial movement but was a property of particles at rest. So Pauli constructed a spin-operator $\mathcal{S} = (\sigma_1, \sigma_2, \sigma_3)$ and marked out σ_3 as a distinguished direction in order to be able to explain the contribution of $\pm\hbar/2$ to the magnetic moment by the eigenvalue of σ_3. It is striking that this operator is nothing else than a set of unit vectors of Euclidean $\mathbf{R}^3$. Is that a mere curiosity of mathematics or does it have a physical significance? In the space of mundane contemplation there is a difference between things which can be used as metaphors of unit vectors (yardsticks) and operations of rotation. Rotation means movement whereas yardsticks do not by necessity involve movement but they may be at rest. In mundane perception a yardstick is not a rotation. In mathematics, however, such a distinction is not obliging. For consider the multivectors of the Pauli algebra $Cl_{3,0}$ which is generated by the units $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ and select the unit vector $\mathbf{e}_1$ of the odd part. Although $\mathbf{e}_1$ is a vector it can carry out rotations. That is, it transposes e. g. the vector $\mathbf{e}_2$ onto the bivector $\mathbf{e}_{12}$. But the formal equivalence of positions and rotations reaches further. Consider the orientation symmetry $\mathbf{D}_4$.[29] There exists a double-group representation of ${}_{\delta}\mathbf{D}_4$ in the Lipschitz group Γ_3 which is generated by the operators

[40] $\sigma' = (1/\sqrt{2})(\mathbf{e}_1 - \mathbf{e}_2)$ $\qquad S_4 = (1/\sqrt{2})(1 + \mathbf{e}_{12})$

Obviously σ' is a vector in the plane $\{\mathbf{e}_1, \mathbf{e}_2\}$ but at the same time it rotates such vectors, for example we can easily verify that

[41] $(\sigma')^{-1}\mathbf{e}_1\sigma' = -\mathbf{e}_2$ $\qquad (\sigma')^{-1}\mathbf{e}_2\sigma' = -\mathbf{e}_1$

which indeed represents the action of the Schönfließ symbol σ' on the basis. Though σ' is a unit vector, it acts on vectors just like any operator of the rotation group $\mathbf{SO}(3)$. This is very important to notice, namely, that it is not necessary to postulate an essential difference between a directed unit of space and operations such as reflections, rotations and transversions. Briefly, directed space units and units of motion are on one level. The old Parmenides has thought about that. He linked motion to rest and vice verse. There is great wisdom in such a consideration.

Consider the Schönfließ symbols $C_2' \equiv C_{12}$, $C_2'' \equiv C_{22}$, $C_2 \equiv C_{32}$. They are $\mathbf{SO}(3)$-rotations of period 2 (flips) about the axis $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$. In the Pauli spinor space S they can be represented by the matrices

[42]

$$\underline{C}_{12} = -\sigma_2\sigma_3 = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = -\mathbf{e}_{23}$$

$$\underline{C}_{22} = \sigma_1\sigma_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \mathbf{e}_{13}$$

$$\underline{C}_{32} = \sigma_1\sigma_2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = \mathbf{e}_{12}$$

[29] More about the dihedral orientation symmetry ${}_{\delta}D_4$ can be found in "The Reconstruction of orientation and Space" and in Schmeikal (1996 pp. 87-90). Its representation in the Lipschitz-group which is used here is discussed in Quality & Quantity 32: 119-154, 1998.

These matrices transform the basis of $\mathbf{R}^2$ as follows

[43] $\underline{C}_{12}: \{\mathbf{e}_1, \mathbf{e}_2\} \rightarrow \{\mathbf{e}_1, -\mathbf{e}_2\}$

$\underline{C}_{22}: \{\mathbf{e}_1, \mathbf{e}_2\} \rightarrow \{-\mathbf{e}_1, \mathbf{e}_2\}$

$\underline{C}_{32}: \{\mathbf{e}_1, \mathbf{e}_2\} \rightarrow \{-\mathbf{e}_1, -\mathbf{e}_2\}$

Being spin-matrices of the double group the $\underline{C}_{i2}$ have period 4 instead of 2 and they possess second values

[44] $_{\delta}\underline{C}_{i2} = -\underline{C}_{i2}$

The Schönfließ symbols $\underline{C}_{iv}$ are linked with the Pauli spin matrices σ_i by the pseudo scalar $j=\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3$, that is

[45]
$$j = \sigma_1\sigma_2\sigma_3 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

[46] $\sigma_1 = j\,\underline{C}_{12}$, $\sigma_2 = j\,\underline{C}_{22}$, $\sigma_3 = -j\,\underline{C}_{32}$

In accordance with the equations [42] — in the classic sense — any successive measurement of two spin components $\sigma_i\sigma_j$ brings on a Schönfließ period-2 rotation of $\mathbf{R}^3$ and a period-4 rotation of the spinor-space *S*. Thereby all possible reversions of basis vectors $\mathbf{e}_i$ are brought forth. The set of all possible reversions and commutations in the plane basis forms a finite group, namely the automorphism group of planar coordinates, the dihedral group $\mathbf{D}_4$. Its Pauli matrices are respectively

[47] $\sigma' = (1/\sqrt{2})(\sigma_{13} - \sigma_{23})$ $\qquad$ $\sigma'' = (1/\sqrt{2})(\sigma_{13} + \sigma_{23})$

$S_4 = (1/\sqrt{2})(1 + \sigma_{12})$ $\qquad$ $(S_4)^{-1} = (1/\sqrt{2})(-1 + \sigma_{12})$

$C_{12} = \sigma_{32}$ $\qquad$ $C_{22} = \sigma_{13}$

$C_{32} = \sigma_{12}$ $\qquad$ E

Each element g possesses a double value $_{\delta}g = -g$, e. g. $_{\delta}C_{12} = -C_{12} = \sigma_{23}$. The elements of this finite group are sufficient to build up fractal geodesics in a planar cellular space-time such that they carry the necessary information about the orientation symmetry of space. They can also incorporate information about the fractal distribution of spin states. To work this out we have to be aware how each operator of the orientation group $_{\delta}\mathbf{D}_4$ acts on the spin eigenstates and how these eigenstates are affected by a total space involution, that is, main involution of the Clifford algebra. First of all notice that 'spin' is a statement concerning the orientation $\{\sigma_1, \sigma_2, \sigma_3\}$ of a local triad of $\mathbf{R}^3$ and not an *'intrinsic angular momentum'*. This old view of the electron is wrong. We have seen in section 8 that the Schönfließ symbols C_{i2} can be represented in $Cl_{3,1}$ in a peculiar way. Then they are indeed quantum-numbers of orientation. In case that their eigenvalues are equal to ±1 they signify quark states. It is therefore that those symbols gain a special importance once we wish to describe fractal states with an isospin. But we are at present working within the frame of the Pauli algebra. Here the C_{i2} act on the orientation (the classical spin states) as follows:

[48] $\underline{C}_{12}$: $\{\sigma_1, \sigma_2, \sigma_3\} \rightarrow \{\sigma_1, -\sigma_2, -\sigma_3\}$

$\underline{C}_{22}$: $\{\sigma_1, \sigma_2, \sigma_3\} \rightarrow \{-\sigma_1, \sigma_2, -\sigma_3\}$

$\underline{C}_{32}$: $\{\sigma_1, \sigma_2, \sigma_3\} \rightarrow \{-\sigma_1, -\sigma_2, \sigma_3\}$

A total space inversion is to be represented by the grade involution in $Cl_{3,0}$ which turns any spin matrix σ_i into $-\sigma_i$. The Mat(2, **C**) matrices $-\sigma_1$, $-\sigma_2$, $-\sigma_3$ can indeed be considered as *second values* of the Pauli matrices σ_1, σ_2, σ_3. These things are important to be noticed because they shall concern the local orientation of fractal path elements.

It is worth mentioning that for some time it seemed problematic to define space inversion on spinors. But these problems have been gone into by Hestenes and could be elaborated in considerable detail for the Pauli algebra by Zeni (1992). I have settled the case for $Cl_{3,1}$ by formulating a theory of orientation based on finite reflection groups (1996). In the Pauli algebra, however, the main involution can simply be represented by the matrix

[49] $${}_{\delta}E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

which in the orientation group ${}_{\delta}\mathbf{D}_4$ represents the double value of unity. The main involution operator ${}_{\delta}E$ as well as the two Schönfließ rotations $\underline{C}_{12}$, $\underline{C}_{22}$ flip the spin matrix σ_3 between its first and second value thereby transforming a spin up- into a spin down eigenstate. Thus, the spin eigenstates ½ and −½ are related by an involution of orientation. That is the most essential thing we have to know about spin in $Cl_{3,0}$. Let me put it that way: Locally a tilt of spin is linked to space involution. This can be regarded as a dynamic element in a fractal space-time. Suppose we would like to construct a *'fractal geodesic wave'* (a denotation coined by El Nashie) which transports a stable spin eigenstate of σ_3 then we would use those operators of ${}_{\delta}\mathbf{D}_4$ for the construction which do not alter the orientation of σ_3. Those operators are E, S_4, $S_4^2 \equiv C_{32}$ and $(S_4)^{-1} \equiv S_4^3$, briefly, the operators of the cyclic subgroup generated by S_4. We may also consider their double values which are obtained from their first ones by multiplying with ${}_{\delta}E$.

There are several ways to construct fractal geodesics in spinor spaces. The first is based on the use of the operators of the cyclic subgroup of $\mathbf{Z}_4 \subset \mathbf{D}_4$. Namely, they can be used in such a way that they generate a fractal cellular trajectory in space with a definite spin σ_3. Generally, we consider spin representations of some finite orientation symmetry G in some spinor space *S*. Next we define a generative sequence $W = \{g_1, g_2, \ldots, g_m\}$ with definite order and and finite lenght. Consider for example

[50] $$W_{PH} = \{E, S_4^3, E, S_4, C_{32}, S_4, E, S_4^3, E\}$$

To construct a Peano-Hilbert fractal we take the unit

[51] $$\xi_0 = (1/\surd 2)(\sigma_1 - \sigma_2)$$

to represent a starting vector for a fractal geodesic. Its meaning is that a trajectory begins its run in diagonal direction $\sigma_1 - \sigma_2$. The generative sequence shall be called a *'generative Clifford word'* for a specific fractal geodesic. By the aid of such *Clifford words* we shall be able to generate the path element of a fractal. In our example with the Clifford word W_{PH} step by step we shall generate a plane geodesic with spin ½ of the Peano-Hilbert type. For this purpose we need the *'single cell orientations ξ_i'* from which we can calculate the locations x_i which form the singularities (corners) of a specific fractal path element. The single cell orientations generated by the Clifford word W_{PH} from the start vector $\xi_0 = (1/\surd 2)(\sigma_1 - \sigma_2)$ are the

[52] $$\xi_i = (1/\surd 3)\,(g_i)^{-1}\,\xi_0\,g_i\,.$$

In detail

[53] $$\xi_1 = (1/\surd 3)\,\xi_0 = (1/\surd 6)\,(\sigma_1 - \sigma_2) \quad \xi_2 = (1/\surd 3)\,S_4\,\xi_0\,S_4^3 = (1/\surd 6)\,(\sigma_1 + \sigma_2)$$

$$\xi_3 = (1/\surd 3)\,\xi_0 = (1/\surd 6)\,(\sigma_1 - \sigma_2) \quad \xi_4 = (1/\surd 3)\,S_4^3\,\xi_0\,S_4 = (1/\surd 6)\,(-\sigma_1 - \sigma_2)$$

$$\xi_5 = (1/\surd 3)(C_{32})^{-1}\,\xi_0\,C_{32} = (1/\surd 6)\,(-\sigma_1 + \sigma_2)$$

$$\xi_6 = (1/\surd 3)\,S_4^3\,\xi_0\,S_4 = (1/\surd 6)\,(-\sigma_1 - \sigma_2)$$

$$\xi_7 = (1/\surd 3)\,\xi_0 = (1/\surd 6)\,(\sigma_1 - \sigma_2)$$

$$\xi_8 = (1/\surd 3)\,S_4\,\xi_0\,S_4^3 = (1/\surd 6)\,(\sigma_1 + \sigma_2)$$

$$\xi_9 = (1/\surd 3)\,\xi_0 = (1/\surd 6)\,(\sigma_1 - \sigma_2)$$

Those single cell orientations define the following path element:

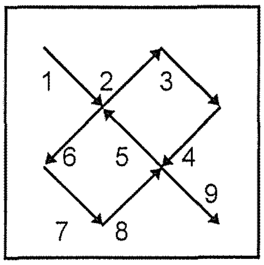


Figure 1: Path element of a Peano-Hilbert fractal

From the single cell orientations we calculate the singularity locations x_i

[54] $x_i = \sum_{\nu=1}^{i} \xi_\nu = \xi_1 + \xi_2 + \ldots + \xi_i$.

in this example:

$x_1 = (1/\sqrt{6})\,(\sigma_1 - \sigma_2)$ $x_2 = (1/\sqrt{6})\,2\sigma_1$ $x_3 = (1/\sqrt{6})\,(3\sigma_1 - \sigma_2)$

$x_4 = (1/\sqrt{6})\,(2\sigma_1 - 2\sigma_2)$ $x_5 = (1/\sqrt{6})\,(\sigma_1 - \sigma_2)$ $x_6 = (1/\sqrt{6})\,(-2\sigma_2)$

$x_7 = (1/\sqrt{6})\,(\sigma_1 - 3\sigma_2)$ $x_8 = (1/\sqrt{6})\,(2\sigma_1 - 2\sigma_2)$ $x_9 = (1/\sqrt{6})\,(3\sigma_1 - 3\sigma_2)$

As can easily be seen from figure 1 those nine vectors represent the locations of the nine singularities of the path element of a Peano-Hilbert fractal geodesic. We say that the x_i constitute the generative path element of a Peano-Hilbert curve. It is therefore that we call the Clifford word $W_{PH} = \{E, S_4^3, E, S_4, C_{32}, S_4, E, S_4^3, E\}$ a generative sequence to a plane Peano-Hilbert geodesic with fractal dimension 2 and spin ½. Namely, none of the operators in the sequence W_{PH} changes the state of spin σ_3.

Next, we can use the generative sequence W_{PH} to construct the second iteration to the Peano-Hilbert geodesic by partitioning each ξ_i further into elementary cell directions:

[55] $\xi_{ij} = (1/3)\,(g_j)^{-1}(g_i)^{-1}\,\xi_0\,g_i\,g_j$.

in this example:

$$\left.\begin{array}{l} \xi_{11} = (1/3)\,(g_1)^{-1}(g_1)^{-1}\,\xi_0\, g_1\, g_1 = (1/3\sqrt{2})\,(\sigma_1 - \sigma_2) \\ \cdot \\ \cdot \\ \cdot \\ \xi_{19} = (1/3)\,(g_9)^{-1}(g_1)^{-1}\,\xi_0\, g_1\, g_9 = (1/3\sqrt{2})\,(\sigma_1 - \sigma_2) \\ \xi_{21} = (1/3)\,(g_1)^{-1}(g_2)^{-1}\,\xi_0\, g_2\, g_1 = (1/3\sqrt{2})\,(\sigma_1 + \sigma_2) \\ \xi_{22} = (1/3)\,(g_2)^{-2}\,\xi_0\, g_2^2 = (1/3\sqrt{2})\,(-\sigma_1 + \sigma_2) \\ \cdot \\ \cdot \\ \cdot \\ \xi_{99} = (1/3)\,(g_9)^{-2}\,\xi_0\, g_9^2 = (1/3\sqrt{2})\,(\sigma_1 - \sigma_2) \end{array}\right]\ \text{81 path elements of the second iteration}$$

Again from these vectors the singularity locations of the second iteration can be calculated

[56] $$x_{ij} = \sum_{\nu,\mu=1}^{i}\sum^{j} (g_\nu)^{-1}(g_\mu)^{-1}\,\xi_0\, g_\mu\, g_\nu$$

Generally, we can define the single cell orientations $\xi_{ij\ldots n}$ by the equation

[57] $$\xi_{ij\ldots t} = 3^{-t/2}\,(g_t)^{-1}\ldots(g_i)^{-1}\xi_0\, g_i g_j \ldots g_t$$

The singularity location with the multiindex ij...n is then

[58] $$x_{ij\ldots t} = 3^{-t/2}\sum_{\nu}^{i}\sum_{\mu}^{j}\sum_{\rho=1}^{t} (g_\rho)^{-1}\ldots(g_\nu)^{-1}\xi_0\, g_\nu\, g_\mu \ldots g_\rho$$

In the example of W_{PH} any index i, j, t runs from 1 to 9. If the multiindex ij... t comprises n components it is running through 9^n values. However, a complete fractal geodesic should be defined by an infinite number of iterations $n \to \infty$. We can define the path X to the n'th iteration as the total set of singularities $X = \{x_{ij\ldots t}\}$ where t is the n'th index component.

Note that any approximation to the order n of a fractal geodesic is a subset of the Clifford algebra, that is, we have $X \subset Cl_{3,0}$. In order to calculate the capacity dimension of the Peano-Hilbert geodesic we use the formula

[59] $$D = \lim_{\varepsilon\to 0} \ln N(\varepsilon)/(-\ln \varepsilon)$$

where $N(\varepsilon)$ is the number of modes needed to cover the fractal. To understand this consider the first iteration. We begin with a vector ξ_0 with unit norm and proceed with the ξ_i (with i=1, 2, ..., 9), each of which has norm $|\xi_i| = 1/3$. Cell orientations to the second iteration have lenght $|\xi_{ij}| = 1/9$ and the norm to iteration n is equal to $|\xi_{ij...t}| = 3^{-n}$ which determines ε. To calculate $N(\varepsilon)$ we must consider the lenght of the Clifford word $|W_{PH}| = 9$. At each step of iteration the number of modes increases by a factor 9. That is, the second iteration comprises $|W_{PH}|^2 = 81$ singularities, . . ., the n'th iteration 9^n.

[60] $$D = \lim_{n\to\infty} \frac{\ln |W_{PH}|^n}{-\ln 3^{-n}} = \lim_{n\to\infty} \frac{n \ln |W_{PH}|}{n \ln 3} = 2n \ln 3 / n \ln 3 = 2$$

Thus a quantum trajectory of the Peano-Hilbert type would have a fractal dimension of 2, which is in correspondence with the results obtained by Feynman. Obviously, such a type of geodesic has a somewhat unrealistic construction principle. But it has been used to demonstrate some of the features of a new concept of space-time and others have used it too. So it may serve as some kind of central example in teaching the properties of new concepts. Going deeper into these matters I have found out that there exists an infinite class of mirror patterns of arbitrary complexity which can be used as fundamental path elements of fractal spinor spaces. Surprisingly, they are more related to the geometric concepts of ethnomathematics than to western images but they seem to comprise all the complexity of quantum geodesics and the related quantum phenomena. For the present, let us only take notice of the fact that it needs a certain number of iterations until the observer 'moves into' the scale of the Planck lenght $L_P = 1{,}61.10^{-33}$. This number is determined by the equation

[61] $3^{-n} = 1{,}61.10^{-33}$ or $n = 69$

Such a *'geodesic wave'* has about 10^{66} cells. The lenght scale corresponding with the classical electron radius is reached after about 27 iterations.

Generally, consider the orientation symmetry ${}_8\mathbf{D}_4 \subset SU(2)$. A fractal geodesic linked with this orientation group can be defined as was outlined in the above procedure:

D1: Let $g_i \in {}_8\mathbf{D}_4$ and $W = \{g_1, \ldots, g_m\}$ a generating sequence of Clifford

numbers with lenght $|W| = m$. Elementary cell orientations are given by

D2: $\xi_i = 3^{-1/2}\, g_i^{-1}\, \xi_0\, g_i$

.

.

.

$\xi_i = 3^{-n/2}\, g_t^{-1} \ldots g_i^{-1}\, \xi_0\, g_i\, g_j \ldots g_t$

where ξ_0 has unit norm and points towards the starting direction. The g_i are double covers of operators of the rotation group ***SO(3)***, which is in agreement with the isomorphy ***SO(3)*** $\simeq$ ***SU(2)***/{±1}. Therefore, provided that W contains any of the operators σ', σ'', C_{12}, C_{22} or ${}_{\delta}E$, the geodesic will sometimes change σ_3, otherwise its spin will be invariant.

We said there existed more than one possibility to construct fractals in spinor spaces. The first, which has just been demonstrated, is based on the use of double covers of orientation groups such as $\mathbf{D}_4$ embedded in the spin-group of a Lipschitz-group. A second weakens this condition and works with isomorphic representations in the Lipschitz-group but outside the spin-group. To see how that goes consider the universal Clifford algebra $Cl_{p,q}$ generated by a real n-dimensional orthogonal space $\mathbf{R}^{p,q}$ of signature (p, q). Let $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$ be a fixed choice of orthonormal units for $\mathbf{R}^{p,q} \subset Cl_{p,q}$. Then under Clifford multiplication the $\mathbf{e}_i$ (together with −1 if n=p=1) generate a finite group G(p, q) called the multivector group of $Cl_{p,q}$ by some (Bergdolt 1996) and Dirac group by others (Shaw 1995). We shall use the denotation of a *'Dirac group'* and define

[62] $G(p, q) = \{\pm 1, \pm \mathbf{e}_i, \pm \mathbf{e}_i\mathbf{e}_j, \ldots, \pm \mathbf{j}\}$ a group of order 2^{n+1}

where $\mathbf{j} = \mathbf{e}_1\mathbf{e}_2 \ldots \mathbf{e}_n$ is the director in $Cl_{p,q}$. Generally the Dirac group is non-abelian because of the anticommutativity of the basis vectors but the quotient group G(p, q)/{±1} by the central subgroup $GF(2) = \mathbf{F}_2 = \{\pm 1\}$ is an elementary abelian 2-group V_n of order 2^n. Note that

[63] $G(1, 1) = G(2, 0) = \{\pm 1, \pm \mathbf{e}_1, \pm \mathbf{e}_2, \pm \mathbf{e}_{12}\} \simeq \mathbf{D}_4$

Now we define monolinear space tracings in $Cl_{p,q}$:

D3: A *monolinear space tracing* is given by a starting vector $\xi_0 \in Cl_{p,q}$; a real number $\alpha<1$ together with a finite generative set $W \subset Cl_{p,q}$ with

[64] $W = \{\omega_i\}$ and $\omega_i \in G(p, q)$ and $i = 1, 2, \ldots, n$

[65] W is called a *loop* if

$$\sum_{r=1}^{n} \prod_{i=1}^{r} \omega_i = 0.$$

D4: Path elements of fractal trajectories are monolinear space tracings which comprise an *elementary orientation set* $\Xi = \{\xi_1, \xi_2, \ldots, \xi_n\}$ given by the recursion relations

[66] $\xi_1 = \alpha\xi_0\,\omega_1,\ \xi_2 = \xi_1\,\omega_2,\ \ldots,\ \xi_r = \xi_{r-1}\,\omega_r,\ \ldots,\ \xi_n = \xi_{n-1}\,\omega_n$

D6: A *Clifford word of lenght r* is implicitly defined by the formula

[67] $W_r = \prod_{i=1}^{r} \omega_i$ with r=1, 2, . . . , n and $F(W) \equiv \{W_r \,/\, r=1, 2, ..., n\}$

So because of [66] we have to have

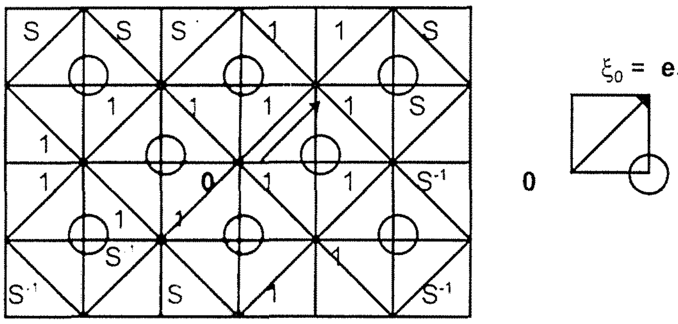
[68] $\xi_r = \alpha \xi_0 W_r$.

D7: Next consider an *elementary route* $X = \{x_1, x_2, . . ., x_n\}$ as given by the relations

[68] $x_r = \sum_{j=1}^{r} \xi_j = \alpha \xi_0 \sum_{j=1}^{r} W_j$ with r=1, 2, . . . , n

Thus the r'th grid point x_r on the route of the fractal path element is given by a sum of multivectors ξ_r. Those are the junctions through which the path element is passing.

Example: path element on the grid R[2,3] starting at **0**
— a monolinear reflection field which is a loop —[30]



Consider the Pauli algebra $Cl_{3,0}$. We begin to read the generative set W at the origin and proceed by following the route as indicated by the starting vector thereby experiencing several reflections. Thus we have a generating set

$W = \{1, 1, S, S, 1, 1, S, 1, 1, S, S, 1, 1, 1, S^{-1}, S^{-1}, 1, 1, S^{-1}, 1, 1, S^{-1}, S^{-1}, 1\}$

where $S = -\sigma_{12} = \sigma_{21}$ and $S^{-1} = \sigma_{12}$. Note that $S, S^{-1} \in G(2, 0)$ and clearly $\in \mathbf{D}_4 \cap_\delta \mathbf{D}_4 \cap_\delta \mathbf{O}_h$.

[30] Note that this represents the image of the Tshokwe sand drawing *'vusamba' on the grid R[2,3] meaning 'friendship'* (see the 'second reconstruction').

$F(W) = \{1, 1, \sigma_{21}, -1, -1, -1, \sigma_{12}, \sigma_{12}, \sigma_{12}, 1, \sigma_{21}, \sigma_{21}, \sigma_{21}, \sigma_{21}, 1, \sigma_{12}, \sigma_{12}, \sigma_{12}, -1, -1, -1, \sigma_{21}, 1, 1\}$

Summing up all elements of F(W) and considering that $\sigma_{12} + \sigma_{21} = 0$, we obtain

$\sum_{r=1}^{n} \prod_{i=1}^{r} \omega_i = \sum_{r=1}^{n} W_r = 0$ that is, W is a loop.

The *elementary orientation set* $\Xi=\{\xi_r\}$ and the route $X=\{x_r\}$ of the monolinear space tracing are given by the expressions

$\xi_0 = \sigma_1+\sigma_2 \qquad \alpha=1$

$\xi_1 = \xi_0\, \omega_1 = (\sigma_1+\sigma_2)1 = \sigma_1+\sigma_2 \qquad x_1 = \sigma_1+\sigma_2$

$\xi_2 = \xi_1\, \omega_2 = (\sigma_1+\sigma_2)1 = \sigma_1+\sigma_2 \qquad x_2 = \xi_1 + \xi_2 = 2\sigma_1+ 2\sigma_2$

$\xi_3 = \xi_2\, \omega_3 = (\sigma_1+\sigma_2)S = (\sigma_1+\sigma_2)\, \sigma_{21} = +\sigma_1-\sigma_2 \qquad x_3 = 3\sigma_1+\sigma_2$

and so on until to $x_{23} = -\sigma_1 - \sigma_2$, $\xi_{24} = \xi_0 = \sigma_1 + \sigma_2$, and $x_{24} = x_{23} + \xi_{24} = 0$, which closes the loop.

A second example is the construction of the Peano-Hilbert fractal, which has been considered by El Naschie (1995) to explain the double-slit phenomena of electron scattering. Let $\alpha=1/\sqrt{3}$; $\xi_0 = (1/\sqrt{2})(\sigma_1-\sigma_2)$ be the start orientation and σ_i the Pauli matrices. Define

[69] $W = \{1, \sigma_{12}, \sigma_{21}, \sigma_{21}, \sigma_{21}, \sigma_{12}, \sigma_{12}, \sigma_{12}, \sigma_{21}\}$

the generating sequence of the Peano-Hilbert fractal. The matrix σ_{12} is the bivector $\sigma_1\sigma_2$ with $(\sigma_{12})^2 = -1$. The sequence of Clifford words $F(W) = \{W_r\}$ is calculated according to definition [67]:

[70] $F(W) = \{1, \sigma_{12}, 1, \sigma_{21}, -1, \sigma_{21}, 1, \sigma_{12}, 1\}$

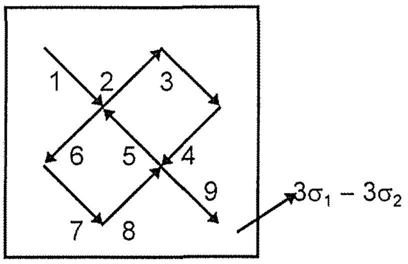
Using formula [68] we calculate the *route* of the Peano-Hilbert tracing:

[71] $\xi_1 = (1/\sqrt{3})\, \xi_0 = (1/\sqrt{6})\, (\sigma_1 - \sigma_2) \qquad \xi_2 = a\, \xi_0 W_2 = (1/\sqrt{6})\, (\sigma_1 + \sigma_2)$
$\xi_3 = a\, \xi_0\, W_3 = (1/\sqrt{6})\, (\sigma_1 - \sigma_2) \qquad \xi_4 = (1/\sqrt{6})\, (-\sigma_1 - \sigma_2)$
$\xi_5 = (1/\sqrt{6})\, (-\sigma_1 + \sigma_2) \qquad \xi_6 = (1/\sqrt{6})\, (-\sigma_1 - \sigma_2)$
$\xi_7 = (1/\sqrt{6})\, (\sigma_1 - \sigma_2) \qquad \xi_8 = (1/\sqrt{6})\, (\sigma_1 + \sigma_2)$
$\xi_9 = (1/\sqrt{6})\, (\sigma_1 - \sigma_2)$

which is the same as in [53]. Observe that

$$\sum_{r=1}^{n} \prod_{i=1}^{r} \omega_i = \sum_{r=1}^{n} W_r = 3$$

that is, W is not a loop but is terminating at the point $x_9 = 3\xi_1 = (1/\sqrt{6})\ (3\sigma_1 - 3\sigma_2)$. In agreement with the well known figure of the Peano-Hilbert path element:



The path element of the Peano-Hilbert fractal geodesic can simply be represented by a symbolic sequence LRRRLLLR, which means that it is built up by one quarter-turn to the left L followed by a quarter-turn to the right followed by a quarter turn to the right and so on. We shall denote this symbolic sequence by the Clifford word

[72] $\quad F_1 = \sigma_{12}\,\sigma_{21}\,\sigma_{21}\,\sigma_{21}\,\sigma_{12}\,\sigma_{12}\,\sigma_{12}\,\sigma_{21} = \phi$

which defines the first iteration of the Peano-Hilbert fractal geodesic. The second iteration is

[73] $\quad F_2 = \phi\,\sigma_{12}\,\phi\,\sigma_{21}\,\phi\,\sigma_{21}\,\phi\,\sigma_{21}\,\phi\,\sigma_{12}\,\phi\,\sigma_{12}\,\phi\,\sigma_{12}\,\phi\,\sigma_{21}\,\phi$

the k'th iteration is given by the recursion equation

.
.
.

[74] $\quad F_{k+1} = F_k\,\sigma_{12}\,F_k\,\sigma_{21}\,F_k\,\sigma_{21}\,F_k\,\sigma_{21}\,F_k\,\sigma_{12}\,F_k\,\sigma_{12}\,F_k\,\sigma_{12}\,F_k\,\sigma_{21}\,F_k$

The scaling factor of the fractal is s=1/3, the number of parts is N=9. Thus, the self-similarity dimension $d_s(F_\infty(s))$ of the fractal $F_\infty(s)$ is equal to

[75] $\quad d_s(F_\infty(s)) = \log N / \log (1/s) = \log 9 / \log 3 = 2.$

When the electron starts off with a definite spin σ_3 this is not altered by the trajectory of $F_\infty(s)$. But in accordance with equation [75] its dimension is 2 instead of 1, which corresponds with the calculation of Feynman (equations [2], [3]). In addition El Nashie (1995, pp. 95) could show that such a Peano-Hilbert geodesic wave or 'Cantorian space-time' obeys the uncertainty relation

[76] $\Delta P \Delta x = (\sqrt{8})h/3 \cong 0{,}942\ h$

where P, x denote impuis and location and h is the Planck constant. Note that the elementary trajectory can be conceived as a reflection field by positing 6 mirrors:

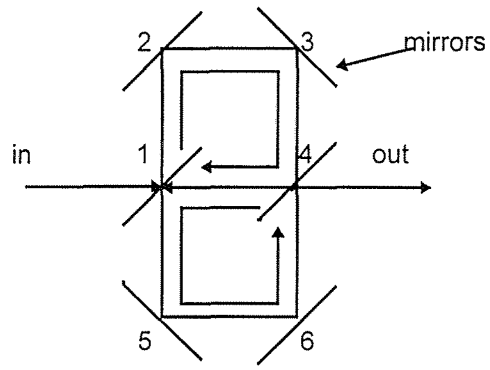


Figure 2: The Peano-Hilbert geodesic wave as a monolinear mirror pattern

This is a very useful construction principle, namely, that any appropriate fractal Cantorian space time can be generated by reflections which constitute its path element.

The Algebraic Infinitesimal Orientation Lattice

Cantorian geometry as a model for quantum space-time has been considered by Ord (1983), Nottale, Le Méhauté and El Naschie (1995) and is based on a work carried out in 1965 by Stenius, who used the Peano curve to map the entire euclidean $\mathbf{R}^2$ on $\mathbf{R}^1$. Similar constructions are possible for one-to-one mappings from $\mathbf{R}$ onto $\mathbf{R}^n$. I do not believe that such a Cantorian space-time provides a realistic model of relativistic quantum physics. Physical reality is more vivid and provides a wider space for ignorance. Nevertheless it may be organized alongside such quasi fractal geodesics. I am convinced that just as no physical trajectory can be an exact straight continuous line also by the same reason it cannot represent exactly a mathematical discontinuous line. In other words mathematical discontinua are not the same as physical discontinua. It is us who use the fractal space-time model as a frame. But it is not nature. Which model will reflect best the physical reality of space-time we do not yet know. We can however be sure that there will not arise an unconquerable technical barrier when it is attemped to formulate the model within a geometric algebra in such a way that the well-known principles of inner and outer symmetry are fulfilled. We need a frame more simple and providing a larger number of degrees of freedom. To obtain such a frame consider the second iteration of the Peano-Hilbert geodesic:

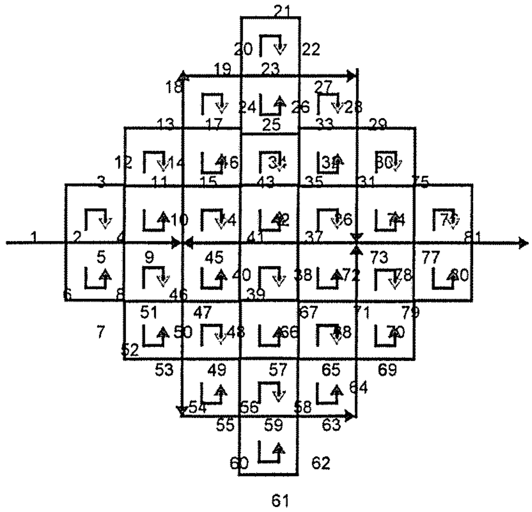


Figure 3: Monolinear route in the second iteration of the Peano-Hilbert fractal

From this figure the building plan of an algebraic lattice can be derived, which provides a better template for dynamics than the concrete fractal itself. Namely, consider the first nine components of the path which correspond to the symbolic series $F_1 = \sigma_{12}\,\sigma_{21}\,\sigma_{21}\,\sigma_{21}\,\sigma_{12}\,\sigma_{12}\,\sigma_{12}\,\sigma_{21}$ of quarter-turns. The route runs from 1 to 81 and comprises all cells of a unit area of the plane. Consider a modulo-4 enumeration of the same route as is displayed in figure 4. You can see that each cell has a definite orientation given by one element of the dihedral group $\mathbf{D}_4$ and has immediate neighbours with the opposite orientation. While a given cell runs clockwise its immediate neighbours run counter-clockwise. Thus the Peano-Hilbert fractal generates an orientation decomposition of the plane. Once the number of iterations $F_k \rightarrow F_{k+1}$ goes to infinity we say that the fractal generates an *infinitesimal orientation decomposition* of the euclidean vector-plane $\mathbf{R}^2$.

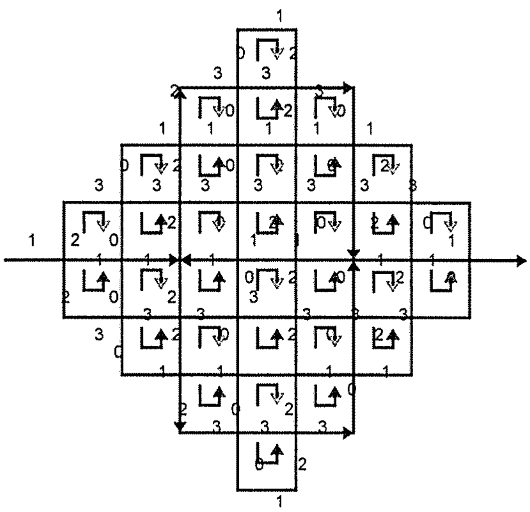


Figure 4: Orientation decomposition of the Peano-Hilbert fractal

There is a peculiar beauty in this lattice structure, namely, each clockwise orientation is compensated by a counter-clockwise rotation. If we think in terms of Clifford algebraic rotation numbers σ_{12}, σ_{21} we find out that each rotation σ_{12} is compensated by a σ_{21}. We count a total of 32 oriented cells and 16 are running clockwise and 16 counter-clockwise. Independent of the generation principle of the lattice we observe the following basic structure which we can denote the algebraic infinitesimal lattice of the euclidean plane:

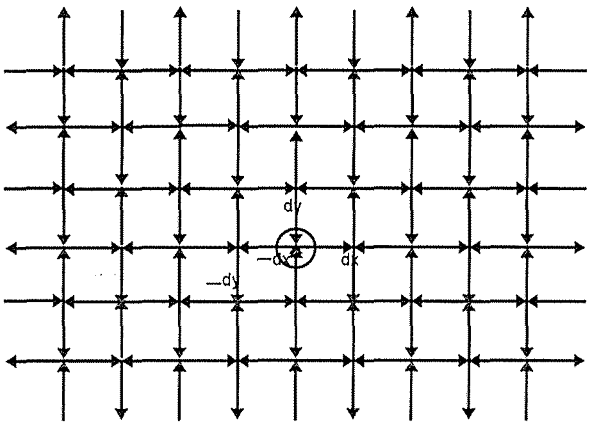


Figure 5: Algebraic Infinitesimal Orientation Lattice (AIOL)

This structure comprises all the important information carried by the Peano-Hilbert geodesic wave. But it is no longer bound to a definite route. It may just as well be derived from any other route or from any other path element as for example from paperfolding sequences (Dekking 1982), dragon curves (Peitgen 1996) and the like. The infinitesimal orientation lattice is a basic template for the representation of discontinuous trajectories just as the euclidean plane is the fundamental representation space for continuous curves. An AIOL allows for the representation of any infinitesimal dynamic element as is represented for example in figure 5:

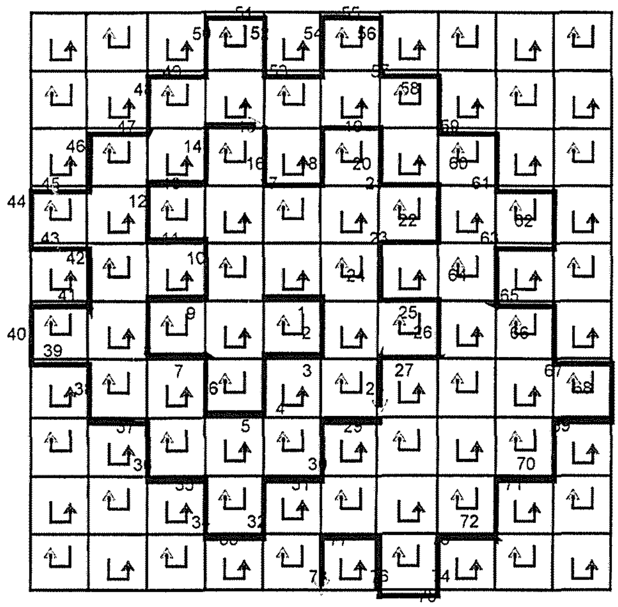


Figure 6: Infinitesimal Diachronic Spheric Motion in the AIOL

The AIOL may just as well be used to represent synchronous spheric wavefronts with a definite spin, and there is no principal technical barrier to formulate the Dirac-Fueter wave equations of matter in discontinuous space-times of Clifford algebraic orientation lattices.

Going back to figure 4, the modulo-4 enumeration of the lattice is in accordance with modulo 4-periodicity of simple and semi-simple Clifford algebras and the central role of the dihedral group as a main factor in the Dirac group of any universal real Clifford algebra. Namely, the Dirac group of any real Clifford algebra $Cl_{p,q}$ can be factorized into a product

[77] $D(p,q) = \mathbf{D_4} \circ \mathbf{D_4} \circ \ldots \circ \mathbf{D_4} \circ \mathbf{G}$

where **G** is either **1** or $\mathbf{Z_4}$, $\mathbf{Q_4}$ or $\mathbf{Z_2 \oplus Z_2}$ or $\mathbf{Q_4 o Z_2 \oplus Z_2}$. Further any Clifford algebra $Cl_{p,q}$ with (p–q) mod 4 $\neq$ 1 is isomorphic with a full matrix algebra over the division ring **R**, **C** or **H**. If (p–q) mod 4 =1 it is semi-simple with two idempotents $(½)(1\pm \mathbf{e}_1\mathbf{e}_2 \ldots \mathbf{e}_n)$ and thus projecting out two copies of a full matrix algebra over one of the division rings **R** or **H**. The factor $\mathbf{D_4} \circ \mathbf{D_4} \circ \ldots \circ \mathbf{D_4}$ in the Dirac group parallels a corresponding factor in the Clifford algebra $Cl_{p,q}$. This is the factor $Cl_{1,1} \otimes Cl_{1,1} \otimes \ldots \otimes Cl_{1,1}$ with a neutral signature (m, m) and each of these factors $Cl_{1,1}$ gives rise to a plane infinitesimal orientation lattice. If we wish to enumerate trajectories in space and color space (see sections 8, 9) the modulo 4-enumeration suffices because of the octahedral orientation symmetry of those spaces which is isomorphic to the symmetric group $\mathbf{S_4}$.

Concluding Comprehension

Nature poses in front of our minds certain invariant structural features of the subnuclear physical world. They are known to us as the *Standard Model* of High-Energy Physics which provides a description of the inner symmetry of the interaction spaces of matter. Those run parallel with some invariant features of physical space-time, which we know as the outer symmetries of macroscopic space-time or Poincaré group. The science historic paths of those two resemble the routes of two utterly different trains running parallel (over the USA or EUROPE), never touching each other yet complaining that the one could not travel without the other. Once we become aware of the geometric algebraic language, which comprises both, it is clear that the Standard Model follows from the observed outer symmetry of the space-time, and observations of a subnuclear world following the Standard Model create a disposition for physical space-times with the observed outer symmetry. This has been shown here by using the geometric Clifford algebra approach as a common language of science. Metaphorically, the so-called *'eightfold path'* is not a property of HEPHY but of the theory of geometric algebras, that is, a metatheory of mathematics. But the present Ansatz goes further inasmuch as it respects both connections between space and time that we ought to be familiar with. The first is what we observe as an expression of special relativity and which connects the measures of space and time. This is well known to each of us. But the second is not well-known. It concerns the calculus of 'relational extension' and the related critics of Alfred North Whitehead as to the usefulness of the method of 'extensive abstraction' in general and relativity theory in particular. Whitehead's insight into the process of nature reaches much further than that of nowaday mathematical physics. It is therefore that I have apprehended Whiteheads philosophy of relational extension. Again it can be shown by algebraic methods that relational extension comprises a symmetry which is basic to the orientation of physical spaces. In this way a language that at first encounter seems to be a mere calculus of temporal procedures of events (extending over and being extended over by other events) and therefore utterly free of any features of orientation and topology, finally leads to the conclusion that any order of real events thus arranged in aggreement with 'extension' comprises geometric orientation. Thus, relations of extension turn out as organizing units for oriented spaces. But measurable space-time goes beyond orientation. In this work it is proposed to formulate equations of motion in the manner of Dirac-Fueter systems, but still leave a space open for the fractal approach. It is shown that it is possible to construct fractal spinor spaces and that trajectories of relativistic subnuclear particles with a spin can have dimensions unequal unity. In this way several disjoint parts of mathematical physics may come together to form a new spin gauge theory. Unresolved for physical theory, however, remains the relation of Hephy with Whitehead's *'awareness'* as the only agent capable to disclose to our minds the features of nature. It seems to have become the only way of nowaday physics to design total concepts, which, as a matter of fact, do not exist. May further endeavours lead us to a more realistic sight. A reconstruction of physics definitely has to reflect upon the heritage of medieval science.

References

Abbott, L. F., Wise, M. B. (1981). Dimension of a quantum mechanical path. *Am. J. Phys.* **49**, 37-39.

Ablamowicz, R., Lounesto, P., Parra, J. M. (1996). *Clifford Algebras with Numeric and Symbolic Computations.* Boston.

Amann, A. (1995). Einige ausgesprochen soziologische Gedanken zum Begriff der Zeit. Unveröffentlichtes Manuskript. Wien.

Amann, A., Schmeikal, B. (1998). *Die Zeit in der Soziologie und in der Physik.* Unveröffentlichtes Vorlesungsmanuskript. Inst. für Soziologie der Univ. Wien.

Ascher, M. (1991). *Ethnomathematics - A multicultural view of mathematical ideas.* Pacific grove.

Ascher, M., D'Ambrosio, U. (1994). *Ethnomathematics: A dialogue, For the Learning of Mathematics.* Vancouver, Vol. 14, No. 2, 36-43.

Assmann, J. (1989). Das Doppelgesicht der Zeit im altägyptischen Denken, in: Aschoff, J., Assmann, J., Blaser, J. P. (Hrsg.), Die Zeit - Dauer und Augenblick. München.

Backmann, G. (1943). Wachstum und organische Zeit, in: Bios, Abhandlungen zur theoretischen Biologie und ihrer Geschichte sowie zur Philosophie der organischen Naturwissenschaften. Leipzig, Bd. 15.

Baumann, H. (1929). *Afrikanische Kunstgewerbe,* in: H. Bossert (Hrg.), Geschichte des Kunstgewerbes aller Zeiten und Völker. Berlin, Bd. 2, 51-148.

Baylis, W. E. (1993). *Electrons, Photons, and Spinors in the Pauli Algebra.* in: Z. Oziewicz et al. (eds.), *Spinors, Twistors, Clifford Algebras and Quantum Deformations.* (97-106). Dordrecht.

Belger, M., Ehrenberg, L. (1981). *Theorie und Anwendung der Symmetriegruppen.* Frankfurt am Main.

Berman, M.(1981). *The reenchantment of the world.* Ithaca, NY.

Berman, M. (1984). *Der Unsichtbare Teil der Vergangenheit.* in: G. Geisler (Hrg.) New Age - Zeugnisse der Zeitenwende. 167. Freiburg im Breisgau.

Bieri, P. (1972). Zeit und Zeiterfahrung. Frankfurt am Main.

Bishop, A. (1989). *Cultural Imperialism and western mathematics: the hidden persuader.* (preprint).

Bishop, A. (1994). Cultural Conflicts in Mathematics Education: developing a Research Agenda. *For the Learning of Mathematics.* Vancouver. Vol. 14, No. 2. 15-18.

Boadella, D. (1991). *Befreite Lebensenergie - Einführung in die Biosynthese.* München.

Born, M. (1939). Reciprocity and the number 137; Part I. *Proc. Roy. Soc. Edinburgh,* 59, 219-223.

Borst, A. (1990). *Computus-Zeit und Zahl in der Geschichte Europas.* Berlin.

Boudon, R (1971). *The Uses of Structuralism.* London.

Bourdieu, P (1976). Entwurf einer Theorie der Praxis. Frankfurt am Main.

Bourdieu P (1993). *Outline of a theory of practice.* (Cambridge studies in social anthropology, 16). Cambridge.

Boyesen G (1994). *Über den Körper die Seele heilen - biodynamische Psychologie und Psychotherapie ; eine Einführung.* München.

Carey, M (1986) *Beads and Beadwork of East and South Africa.* Buckinghamshire.

Chisholm J S R (1992). *Tetrahedral structure of idempotents of the Clifford algebra* $Cl_{3,1}$. in A Micali et al (eds.), *Clifford Algebras and their Applications in Mathematical Physics* 27-32, Dordrecht.

Clifford, W K (1882). On the Space-Theory of Matter. in: R. Tucker (ed.), *Mathematical Papers by William Kingdon Clifford.* London.

Cnops, J (1996). *Vahlen Matrices for non-definite metrics.* in: R. Ablamowicz, P. Lounesto, J. M. Parra (eds.), *Clifford Algebras with Numeric and Symbolic Computations.* 155-164. Boston.

Cohen, G. A. (1979). *Karl Marx's Theory of History, a Defence.* Oxford.

Crump, T. (1990). *The Anthropology of number.* Cambridge.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics.* Montreal. Vol. 5, No. 1. 44-48.

D'Ambrosio, U. (1993). Ethnomatemática: um programma, *A educacao matemática em revista.* Blumenau. Vol. 1, No. 1. 5-11.

Dekking, F. M. (1982). Folds I, II, III, *Math. Intelligencer, **4**, 4, 190-195.*

Deleuze, G. (1992). *Woran erkennt man den Strukturalismus?* Berlin.

Dutt, B. B. (1925). *Town Planning in Ancient India.* Calcutta and Simla.

Dux, G. (1992). Die Zeit in der Geschichte. Frankfurt am Main.

Eddington, A. S. (1935). *New Pathways in Science.* 271. Cambridge.

Eddington, A. S. (1936) *Relativity Theory of Electrons and Protons.* 36. Cambridge.

Elias, N. (1984) Uber die Zeit. Arbeiten zur Wissenssoziologie II. Frankfurt am Main.

El Naschie, M S (1995) *Young double-slit experiment, Heisenberg uncertainty principle and correlation in Cantorian space-time.* in: M. S. El Naschi, O. E. Rossler, I. Prigogine et al. (eds.), *Quantum Mechanics, Diffusion and Chaotic Fractals.* 93-99. Oxford.

Ewers, M (1988) *Zeitordnung des Lebendigen.* in: R. Zoll (Hrg.) *Zerstörung und Wiederaneignung von Zeit.* Frankfurt am Main.

Ferreira, E (1989) *The genetic principle and the ethnomathematics.* in: C. Keitel. 110-111.

Feynman, R P , Hibbs A R (1965). *Quantum Mechanics and Path Integrals.* New York.

Figal, G (1992) *Zeit und Identität - Systematische Überlegungen zu Aristoteles und Platon.* in: Forum fur Philosophie Bad Homburg (Hrg.) *Zeiterfahrung und Personalität.* Frankfurt.

Flusser, V. (1992) *Die Schrift - Hat Schreiben Zukunft?* Frankfurt am Main.

Fontinha, M. (1983). *Desenhos na areia dos Quiocos do Nordeste de Angola.* IICT. Lisboa.

Fraser, J. T. (1991). *Die Zeit.* München.

Frobenius, L. (1939). *Monumenta Afrikana. Der Geist eines Erdteils.* Schriften zur Kulturkunde III. Weimar.

Frutiger, A. (1996). *Der Mensch und seine Zeichen. Schriften, Symbole, Signate, Signale.* Wiesbaden.

Gerber, H. (1952). *Über die Quellen und verfassungsrechtliche Deutung der Mittelalterlichen Quatuorvirate und den geschichtlichen Wert der Vier-Grafen-Würde. Festschrift Edmund E. Stengel.* Münster und Köln. 453-470. (S. 463).

Gerdes, P. (1982). *Mathematics for the benefit of the people.* CARIMATH, Paramaribo, reprint.

Gerdes, P. (1985). Conditions and strategies for emancipatory mathematics education in underdeveloped countries, *For the learning of mathematics.* Montreal, Vol. 5, No. 3, 15-20.

Gerdes, P. (1985). *Zum erwachenden geometrischen Denken.* UEM, Maputo, preprint.

Gerdes, P. (1989). Reconstruction and extension of lost symmetries: examples from the Tamil of South India. *Computers and Mathematics with Applications.* New York. Vol. 17, No. 4-6, 791-813.

Gerdes, P. (1997). *Ethnomathematik - dargestellt am Beispiel der Sona Geometrie.* Heidelberg.

Geroch, R. (1968). Spinor structure of space-times in general relativity I. *J. Math. Phys.* **9**, 1739-1744.

Gibbons, G. W. (1993). *The Kummer configuration and the geometry of Majorana Spinors.* in: Z. Oziewicz et al. (eds.), *Spinors, Twistors, Clifford Algebras and Quantum Deformations.* (39-52). Dordrecht.

Giddens, A. (1984). The Constitution of Society. Cambridge 1984 (deutsch: Die Konstitution der Gesellschaft. Frankfurt am Main.

Goffman, E. (1961). *Asylums.* Harmondsworth.

Goffman, E. (1972) *Interaction Ritual.* London.

Goffman, E. (1981). *Forms of Talk.* Oxford.

Greider, K., Weiderman, T. (1988). *Generalised Clifford algebras as special cases of standard Clifford algebras.* I'UCD Preprint 16.

Grüsser, O-J. (1989). *„Zeit und Gehirn. Zeitliche Aspekte der Signalverarbeitung in den Sinnesorganen und im Zentralnervensystem"*, in: J. Aschoff, J. Assmann, J. P. Blaser et al. (eds.). *„Die Zeit - Dauer und Augenblick".* S. 79. München.

Hamilton, W. R. (1853). *Lectures on Quaternions.* Dublin.

Hestenes, D., Sobczyk, G. (1984). *Clifford Algebra to Geometric Calculus. A Unified Language for Mathematics and Physics.* Dordrecht.

Jadczyk, A. (1993). *Born's reciprocity in the conformal domain.* in: Z. Oziewitcz et al. (eds.), *Spinors, Twistors, Clifford Algebras and Quantum Deformations.* 129-140. Dordrecht.

Keitel, C., Damerow, P., Bishop, A., Gerdes, P. (1989) (eds.). *Mathematics, Education and Society.* UNESCO. Paris.

Keyser, C. (1932). Mathematics as a Culture Clue. *Scripta Mathematica,*Vol. 1, 185-203.

König, M. E. P. (1981). *Am Anfang der Kultur - Die Zeichensprache des frühen Menschen.* Frankfurt am Main.

Korth, L. (1888). *Der Kölnische Bauer und das Quaternionen -System.* Mitteilungen aus dem Stadtarchiv zu Köln. Heft 14. 117-124.

Kubik, G. (1987). African space/time concepts and the tusona ideographs in Luchazi culture with a discussion of possible cross-parallels in music. *African Music.* Grahamstown. Vol. 6, No. 4. 53-89.

Lacan, J. (1973). *Schriften I.* Freiburg im Breisgau.

Laing, R. D. (1971). *Self and Others.* Harmondsworth.

Laing, R. D. (1975). *The Politics of Experience and The Bird of Paradise.* (first publ. 1967) Harmondsworth.

Laing, R. D. (1976). *The Facts of Life.* New York.

Laing, R. D. (1981). *Es stört mich nicht, ein Mensch zu sein.* Köln.

Laughrens, M. (1982). *Walpiri kinship structure.* in: J. Heata, F. Merlan, A. Rumsey (eds.) *Languages of Kinship in Aboriginal Australia.* Sidney.

Le Méhauté, A. et al. (1995). *Introduction to Wave Phenomena and Uncertainty in a Fractal space–I.* in: M. S. El Naschie, O. E. Rossler, I. Prigogine et al. (eds.), *Quantum Mechanics, Diffusion and Chaotic Fractals.* 93-99. Oxford.

Leroi-Gourhan, A. (1964). *Les Religions de la Prehistoire.* Paris

Lipschitz, R. (1886). *Untersuchungen über die Summen von Quadraten.* Bonn.

Lounesto, P., Latvamaa, E. (1980). Conformal transformations and Clifford algebras. *Proc. Am. Math. Soc.*, 79, 533-538.

Lounesto, P., Mikkola, R., Vierros, V. (1987). *CLICAL User Manual - Complex Number, Vector Space and Clifford Algebra Calculator for MS-DOS Personal Computers.* Helsinki.

Lounesto, P., Springer, A. (1989). *Möbius Transformations and Clifford Algebras of Euclidean and Anti-Euclidean Spaces.* in: J. Lawrynowicz (ed.), *Deformations of Mathematical Structures.* 79-90. Dordrecht.

Lounesto, P. (1996). *Verifying and falsifying conjectures. Counterexamples in Clifford algebras with CLICAL.* in: R. Ablamowicz, P. Lounesto, J. M. Parra. (eds.). *Clifford Algebras with Numeric and Symbolic Computations.* 1-30. Boston.

Luquet, G. (1929). Sur l'origine des notions mathématiques: remarques psychologiques et ethnographiques. *Journal de Psychologie,* Paris, 733-761.

Maks, J. (1989). *Modulo (1, 1)-periodicity of Clifford algebras and generalized (anti-)Möbius transformations.* Proefschrift. Delft University of Technology.

Meriggi, P. (1971). *La Scrittura Proto-Elamica.* Accademia Nazionale Dei Lincea. Parte la: La scrittura e il contenuto dei testi. Roma.

Micali, A. et al. (1992). *Clifford Algebras and their Applications in Mathematical Physics.* Dordrecht.

Mohr, G. (1992). *Thesen über Zeitbewußtsein und innere Erfahrung.* in: Forum für Philosophie Bad Homburg (Hrg.) *Zeiterfahrung und Personalität.* Frankfurt.

Müller, W. (1961). *Die Heilige Stadt.* Stuttgart.

Newman (ed.) (1956). *The World of Mathematics.,* New York, Vol. 4, 2348-2364.

Nottale, L. (1995). *Scale relativity, fractal space-time and quantum mechanics.* In: M. S. El Naschi, O. E. Rossler, I. Prigogine et al. (eds.). *Quantum Mechanics, Diffusion and Chaotic Fractals.* 51-78. Oxford.

Ord, G. N. (1983). Fractal space-time: a geometric analogue of reiativistic quantum mechanics, *J. Phys. A: Math. Gen. 16, 1869-1884.*

Pearson, K. (ed.) (1955). *The Common Sense of the Exact Sciences - by William Kingdon Clifford.* New York.

Peitgen, H.-O., Skordev G. (1996). *Falten und Fraktale.* in: R. Hegselmann, H.-O. Peitgen (Hrg.). *Modelle Sozialer Dynamiken - Ordnung, Chaos und Komplexität.* 19-42.

Petraschen, M. I., Trifonow, E. D. (1969). *Anwendung der Gruppentheorie in der Quantenmechanik.* Leipzig.

Piaget, J. (1957). *Logic and Psychology.* New York.

Piaget, J. (1981). *Jean Piaget über Jean Piaget.* München.

Pinxten, R., I. v. Dooren & Harvey F. (1983). *The anthropology of space: exploration into the natural philosophy and semantics of the Navajo.* Pennsylvania.

Prigogine, I., Stengers, I. (1981). *Dialog mit der Natur. Neue Wege Naturwissenschaftlichen Denkens.* München-Zürich.

Raju, C. K. (1994). *Towards a Consistent Theory of Time.* Dordrecht.

Raum, O. (1938). *Arithmetics in Africa.* London.

Reich, W. (1948). *The discovery of the orgone.* New York : Orgone Inst. Pr.

Reich, W. (1984). *Die bio-elektrische Untersuchung von Sexualität und Angst.* Frankfurt am Main.

Reich, W. (1986). *Die Massenpsychologie des Faschismus.* Köln.

Reich, W. (1995). *Menschen im Staat.* Basel.

Rugh, S. E. (1995). *Chaos in the Einstein equations.* in: M. S. El Naschi, O. E. Rossler, I. Prigogine et al. (eds.). *Quantum Mechanics, Diffusion and Chaotic Fractals.* 161-183. Oxford.

Sacks, H., Schegloff, E. A. (1974). A simplest systematics for the organization of turn- talking in conversation. *Language.* Vol. 50.

Schmeikal-Schuh, B. (1989). *Soziogene der Orientierung - Ikonen der Raumzeit.* WISDOM. Jg. III, 3/4. 1-18. Wien.

Schmeikal-Schuh, B. (1993). *Logic from Space.* Quality and Quantity 27. 117-137. Dordrecht.

Schmeikal, B. (1993). *Space-Time Sociology.* Research-Memorandum 313. Institute for Advanced Studies. Vienna.

Schmeikal, B. (1996) *The Generative Process of Space-Time and Strong Interaction. Quantum Numbers of Orientation.* in: R. Ablamowicz, P. Lounesto, J. M. Parra (eds.) Clifford Algebras with Numeric and Symbolic Computations. 83. Boston.

Schmeikal, B. (1997). Review to 1 466 962: Gerdes, Paulus: Ethnomathematik. Spektrum Akad. (1997) *Mathematical Reviews.* forthcoming.

Schütz, A., Luckmann, Th. (1973). *The structures of the life-world.* Evanston, Ill.

Schütz, A..; Parsons, T. (1978). *The theory of social action - the correspondence of Alfred Schütz and Talcott Parsons.* Grathoff, Richard [eds.] Bloomington, Ind.

Schütz, A. (1981). Theorie der Lebensformen (frühe Manuskripte aus der Bergson-Periode) Frankfurt am Main.

Schütz, A. (1993). *Der sinnhafte Aufbau der sozialen Welt - eine Einleitung in die verstehende Soziologie.* Frankfurt am Main.

Schütz, A. (1975-96). *Collected papers. Vol 1 - 4* .The Hague.

Schweber, S. S. (1986). *Feynman and the visualization of space-time processes. Rev. Mod. Phys. 58.* 449-508.

Sethe, K. (1964). *Vom Bilde zum Buchstaben. Untersuchungen zur Geschichte und Altertumskunde Ägyptens - Begründet von Kurt Sethe.* (H. Kees Hrsg.) Bd.12. Hildesheim.

Shan, S., Baily, P. (1991). *Multiple factors: classroom mathematics for equality and justice.* Staffordshire.

Struik, D. (1942). On the Sociology of Mathematics. *Science and Society,* Vol. 6, 58-70.

Stewart, J. (1993). *Advanced General Relativity.* Cambridge.

Stroyan, K. D., Luxemburg, W. A. J. (1976). *Introduction to the Theory of Infinitesimals.* New York.

Taton, R. (1957). *A General History of Sciences.* London.

Tucker, R. (ed.) (1882). *Mathematical Papers by William Kingdon Clifford.* London.

Urry, J. (1996). *Sociology of Time and Space.* in B.S. Turner (ed.) *The Blackwell Companion to Social Theory. Cambridge.*

Vahlen, K. Th. (1902). Ueber Bewegungen und complexe Zahlen. *Mathematische Annalen.* **LV**, 585-593.

Wheeler, J. A., Patton, C. M. (1977). *Is Physics legislated by Cosmogony?* in: R. Duncan, M. Weston-Smith, (eds.), *The Encyclopaedia of Ignorance.* 19-35. Oxford.

Whitehead, A. N. (1964). *The Concept of Nature.* Cambridge.

White, L. (1947). *The locus of mathematical reality*: an anthropological footnote, in: J. Newman 1956

Wilder, R. (1950). The cultural basis of mathematics. *Proceedings Internat. Congress of Mathematicians,* Vol. 1, 258-271.

Wilson, E. (1984). *North Americal Indian Designs.* British Museum. London.

Zoll, R. (1988). *Zeiterfahrung und Gesellschaftsform.* In: ders., *Zerstörung und Wiederaneignung von Zeit.* Frankfurt am Main.

Institut für Höhere Studien
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna
Austria

Phone: +43-1-599 91-216
Fax: +43-1-599 91-191